# From Continuous Dynamics to Discrete Expressions of Behaviour

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# How to bring together:

Observed Expressions of Behaviour/Emotion
(Discrete)

Underlying/accompanying physiological dynamics
(Continuous)

**?**

# Issues

- Empirical

  How to combine the values of discrete and continuous variables in one (statistical) analysis?

  - Continuous Time (Hidden) Markov Chains[1,2]
  - ANOVA to test for the effects of (experimental) conditions on the transition rates[2]
  - Proportional Hazard Models to estimate the effects of various factors on the termination rate of behavioural acts[2]
  - Logistic regression
  - etcetera …

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[1] The use of Markov models will be outlined in the workshop of the summer school

[2] See: Haccou, P. & E. Meelis (1992): Statistical Analysis of Behaviour. Oxford University Press

# Issues

- Theoretical

How to combine discrete and continuous variables conceptually?

How to bring together (continuous) changes in physiology, motivation/emotion and (discrete) expressions of behaviour?

# Overview

1. Models of Behaviour and Motivation

   a. Conventional Approaches
      and some of their drawbacks
   b. Dynamical Systems Modelling
      - a worked out example –

2. Behaviour and Sensor-Motor Coordination

3. Motivation and Emotion driving Behaviour

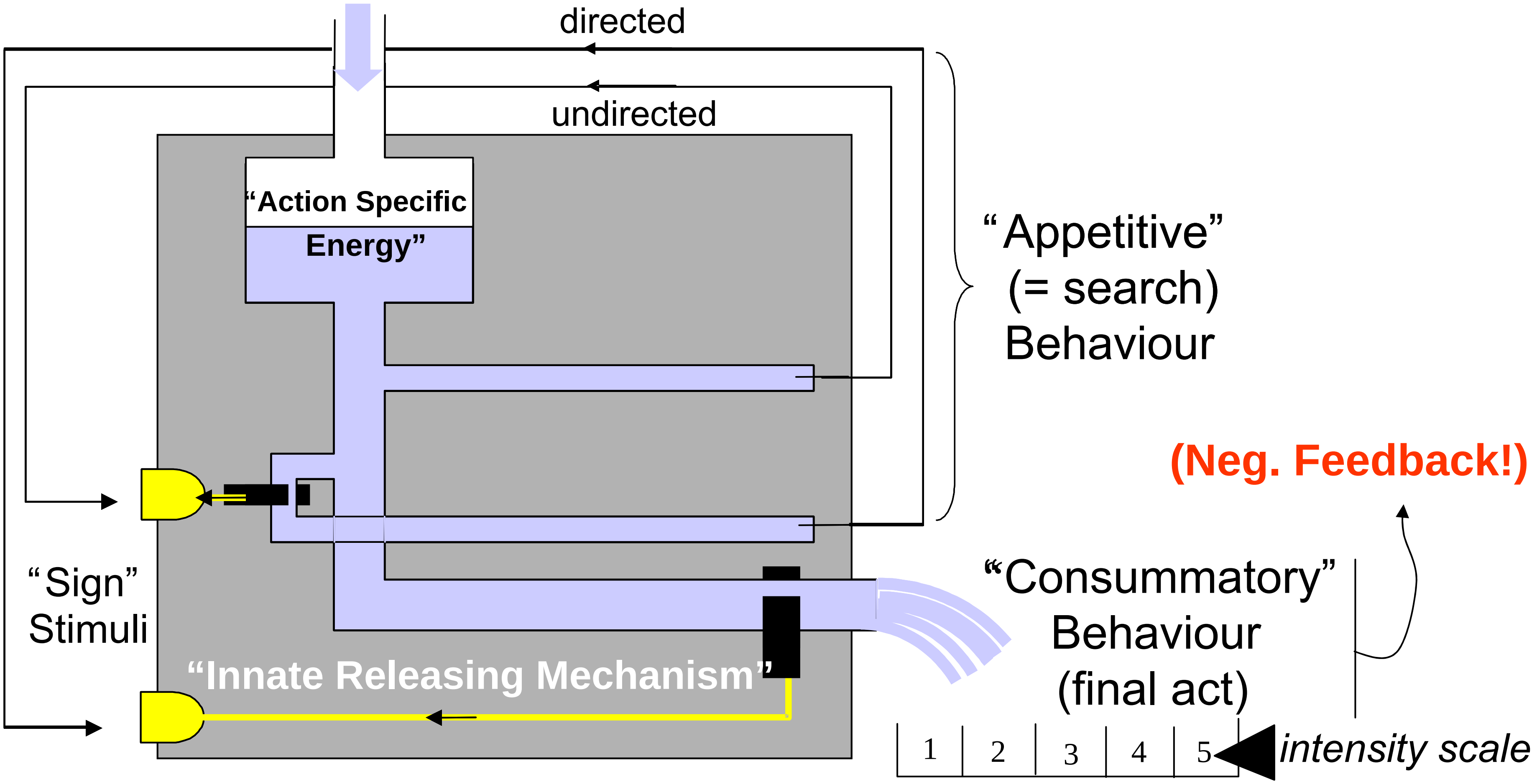
# 1 Models of Behaviour:

## Conventional approaches

When certain internal (physiological / motivational) variables surpass a threshold, a particular (behavioural) activity will be expressed

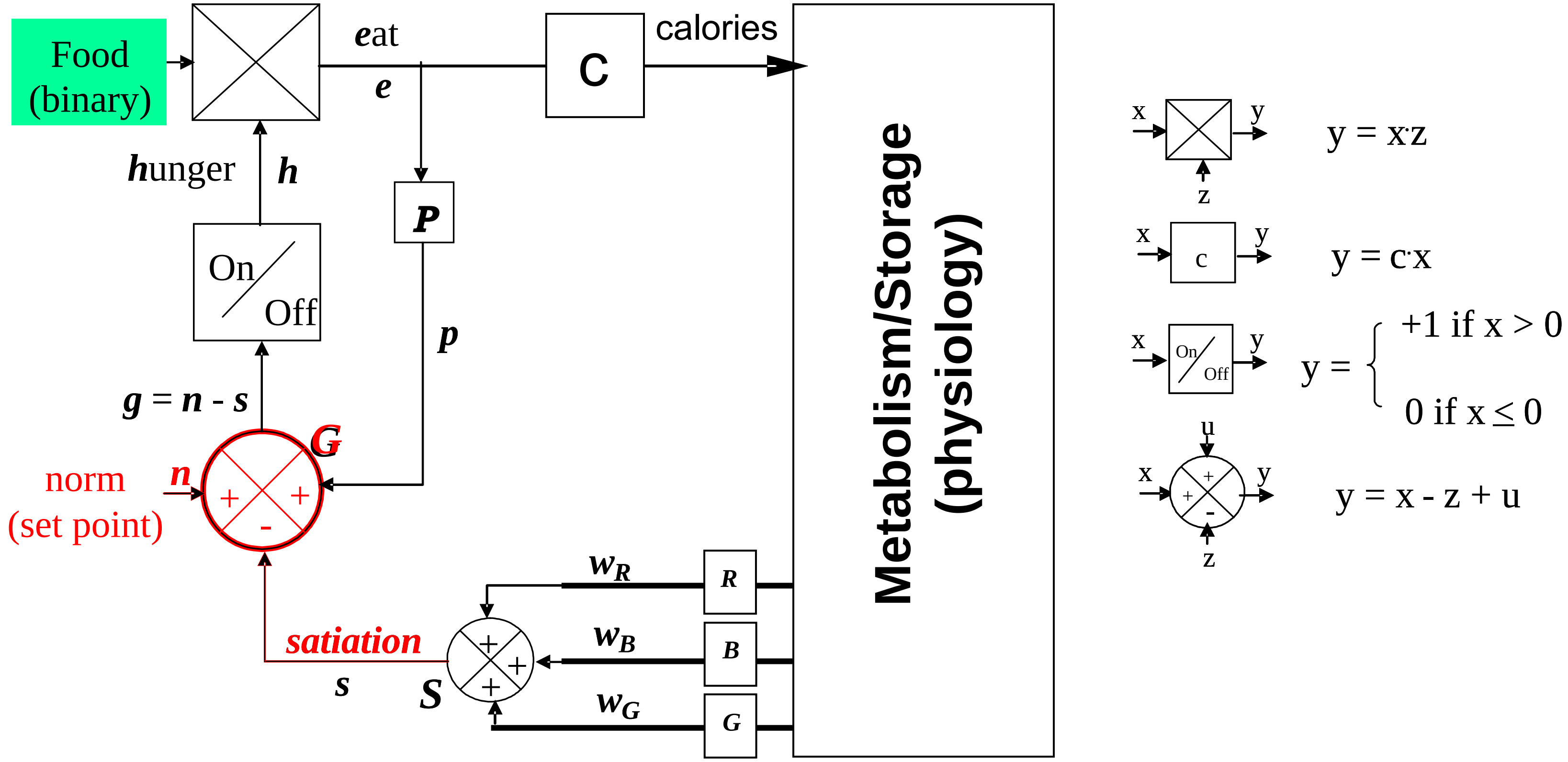
- Classical Ethology
- Cybernetics
- (Functionalist) State Space Approach

# Classical Ethological Approach
# (“Drive”) (Lorenz, Tinbergen)



**Task: Locate “motivation”/”emotion” in this diagram**

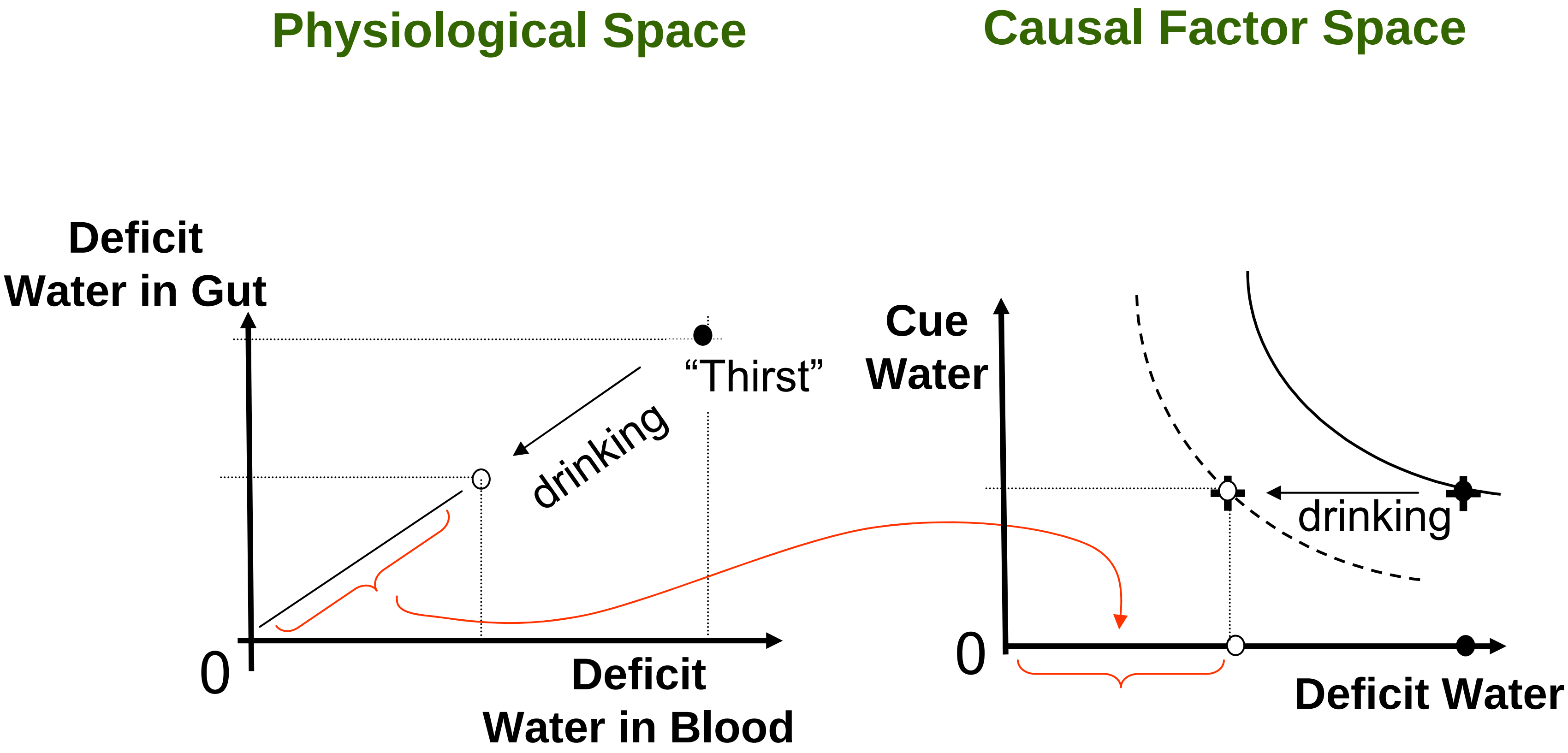
# More formal: **Cybernetics**



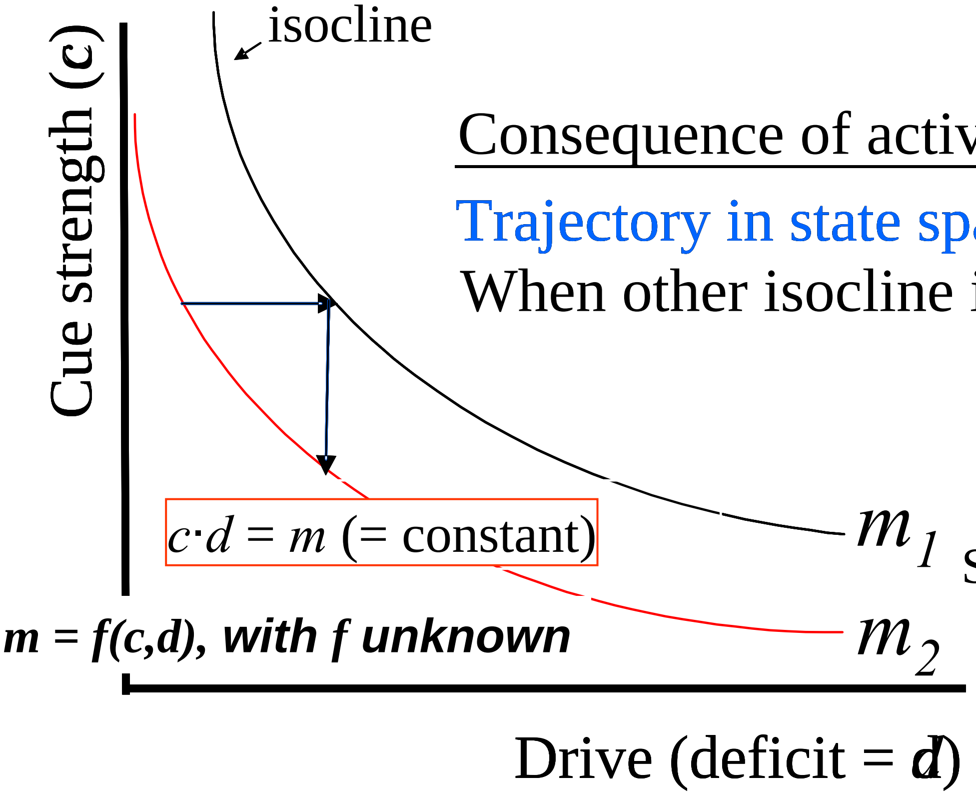
**Task: Locate “motivation”/”emotion” in this diagram**

# Functionalist State Space Approach (McFarland)

## the concept of State Space



# State Space Approach
# (ctd)



Consequence of activity:

Trajectory in state space

When other isocline is cut: changed tendency

Strength of tendency of activity

~Intensity behaviour

***Actually,* $m = f(c,d)$*, with f unknown***

**Task: Locate “motivation”/”emotion” in this diagram**

# Problems of Conventional Approaches

- Basic elements of these models (IRM, ASE, set points, thresholds etc.) are ***hypothetical*** constructs and arbitrary "magic numbers" .
  Only justified because they are demanded by the theory

- "Contemplative" approach that models how we ***think*** about behaviour/emotion rather than behaviour itself

- What are the "real" physiological/motivational variables and how are they interrelated/co-regulated?

# Problems of the Functionalist Approach

1. “Cue” and “Drive” (Deficit) are considered to be independent

2. No formulation of the “rules” underlying the movement in state space

3. Optimisation approach: mainly concerned with the “costs” and “benefits” (in terms of fitness) of the behaviour that reduces the deficits

(Contemplation and arbitrariness!)

# This Talk

1) Limiting the use of magic set points and contemplative gadgets

↓

2) Instead: model in which these features ***arise*** as a result of systems dynamics

↓

3) Bringing together a dynamical systems model of continuous changes in internal variables, sensor input and observable, discrete behaviour in an autonomous agent (robot)

# 2 A Dynamical Systems Approach

**A mathematical formulation:**

- of how self-regulating changes in behavioural tendency, by integrating sensory information and physiological deficits, drive behavioural expression

- that is general enough to bring together some fundamental ideas about motivation and behaviour from (classical) ethology, (neuro-) physiology and psychology

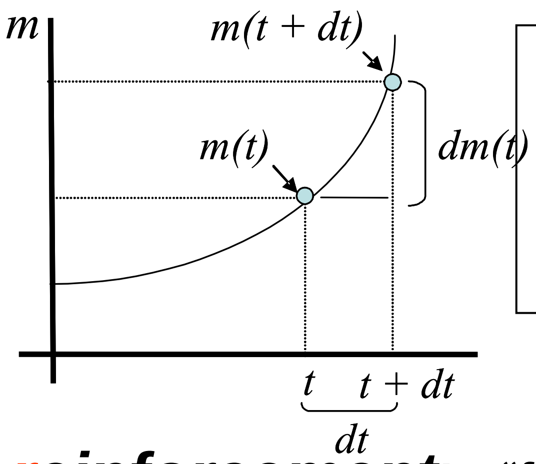
- of a system of which the derived mathematical properties point to relationships between phenomena that were hitherto considered to be unrelated and hence helps in hypothesis building and conceptualising “emotion”

**Heuristic!**

# A (very simple) Model for the Dynamics of Behavioural Tendencies

$m(t)$ = behavioural tendency at time $t$:

* If nothing else happens, it **d**ecays as: $m'(t) = -d\cdot m(t)$



$$m'(t) = \dot{m} = \frac{dm(t)}{dt}$$

change of $m$ over a time interval $dt$

* Grows by ***reinforcement***: "fear breeds fear" $m'(t) = r\cdot m(t)$

$$\boxed{m'(t) = r\cdot m(t) - d\cdot m(t)}$$

# Behaviour tendency both **needs** and **consumes** resources (e.g. **Energy**)

**= *m*obilisation of resources**

- *Resources are used in proportion to behavioural tendency*

$$R_{used} = c \cdot m(t)$$

- *Amount of Resources left over at time t:*

$$\underset{\substack{\uparrow \\ \textit{baseline level}}}{R_0} - R_{used} = R_0 - c \cdot m(t)$$

behaviour tendency is ***reinforced*** by the resources available at time t:

$$m' = \overbrace{(R_0 - R_{used})}^{r} m$$

$$= (R_0 - cm)m$$

# Behaviour tendency increases in proportion to a ***fraction*** ( $f$ ) of the Resources left over:

$$m' = f(R_0 - c\cdot m)m - dm \qquad (0 < f \leq 1)$$

$$= \underbrace{(fR_0 - d)}_{k_1}\cdot m - \underbrace{f\,c}_{k_2}m^2$$

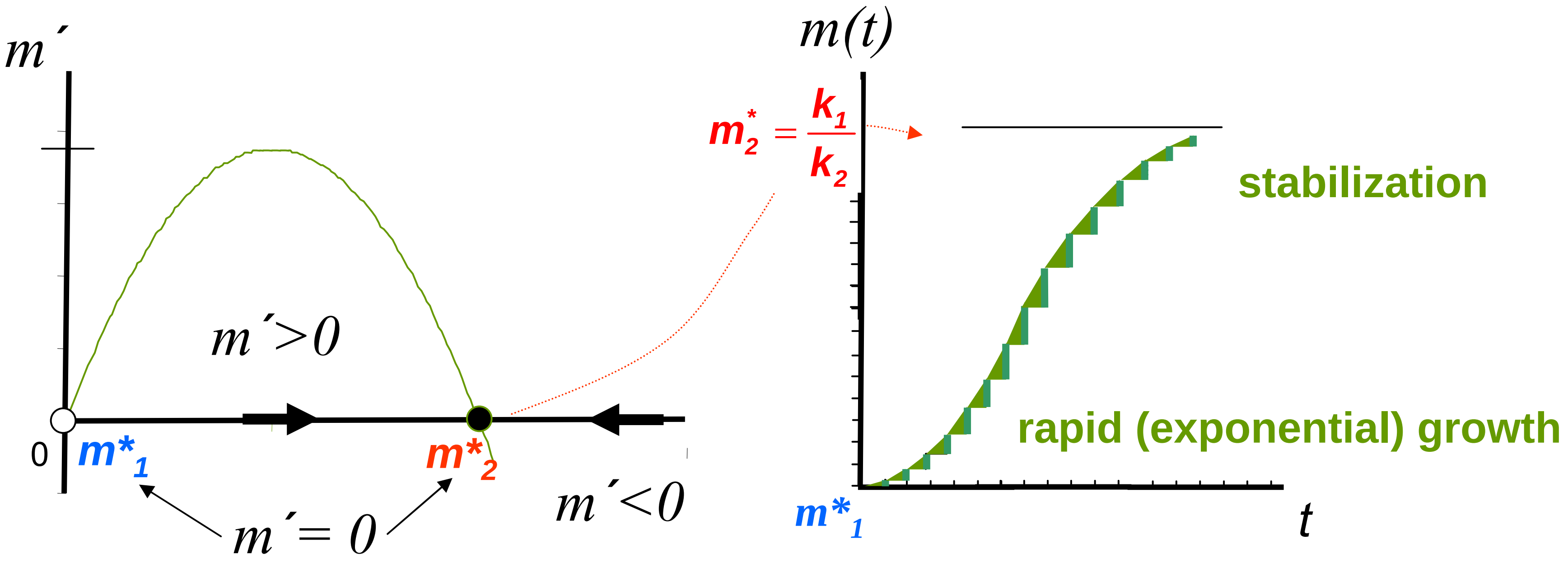
$$m' = k_1 m - k_2 m^2$$

Logistic differential equation

cf "$y = ax - bx^2$"

parabola

# Logistic Behavioural Dynamics



**unstable fixed point**
**(“Repellor”)**

**stable fixed point**
**(“Attractor”)**

$$m' = k_1 m - k_2 m^2 = m\left(k_1 - k_2 m\right)$$

$$m' \; is \; zero \; for: m_1^* = 0, m_2^* = \frac{k_1}{k_2}$$

***Intensity/tendency is fuelled by resources that are diminished by its own increase***

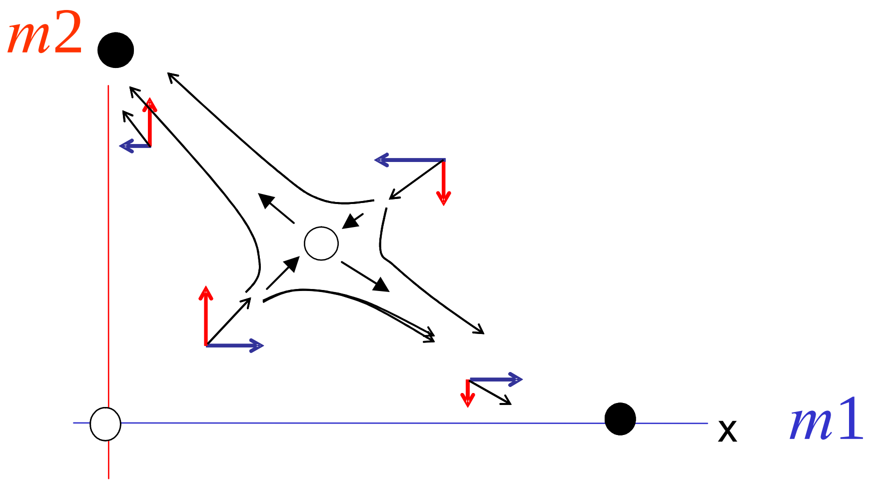
## EXTENSION 1

- When TWO (or more) activities are using up the resource

$$m1' = (R_0 - c_1 m1 - c_2 m_2) m1 - d_1 m1$$

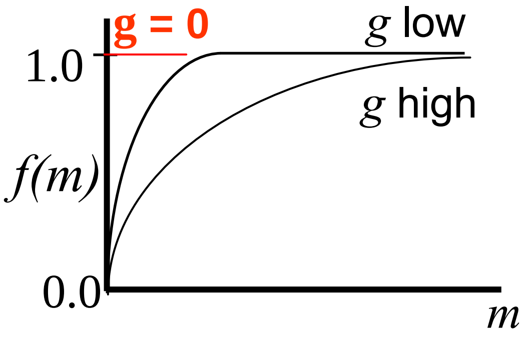
$$m2' = (R_0 - c_2 m2 - c_1 m_1) m2 - d_2 m2$$

***Lotka-Volterra system for competitive exclusion***



Dynamic model for “Winner-Takes-All” action selection

# **EXTENSION 2:** When tendency is stronger, a ***larger fraction*** of available Resources is used to fuel further increase



$$f(m) = \frac{m}{m+\enclose{circle}{g}}$$

Guarantees $\underset{g=\infty}{\underset{\uparrow}{0}} < f(m) \le \underset{g=0}{\underset{\uparrow}{1}}$

$$\downarrow$$

$$m' = f(m)(R_0 - cm)m - dm$$

$$\downarrow$$

$$m' = \frac{m}{m+g}(R_0 - cm)m - dm$$

$$= \frac{-cm^3 + (R_0 - d)m^2 - dgm}{m+g}$$

***Cubic differential equation***

Suggestion:

$$g = \frac{1}{Deficit}$$

**Larger Deficit**:
faster recruitment
of reinforcing
resources

# Dynamics of the System:

## effects of the parameters $g$ and $R_0$

$$m' = \frac{-cm^3 + (R_0 - d)m^2 - dg \cdot m}{m + g}$$

$$= m\left(\frac{-cm^2 + (R_0 - d)m - dg}{m + g}\right)$$

- Equilibrium at $m´ = 0$ = Intersection horizontal axis

  a) $m_1{}^* = 0$

  b) $[-cm^2 + (R_0 - d)m - dg] = 0$

  quadratic equation with roots:

$$m_2^* = \frac{(R_0 - d) - \sqrt{(R_0 - d)^2 - 4cdg}}{2c}$$

$$m_3^* = \frac{(R_0 - d) + \sqrt{(R_0 - d)^2 - 4cdg}}{2c}$$

# Effect of $g$

$$m_1^* = 0$$

$$m_2^* = \frac{(R_0 - d) - \boxed{\sqrt{(R_0 - d)^2 - 4cdg}}}{2c}$$

$$m_3^* = \frac{(R_0 - d) + \boxed{\sqrt{(R_0 - d)^2 - 4cdg}}}{2c}$$

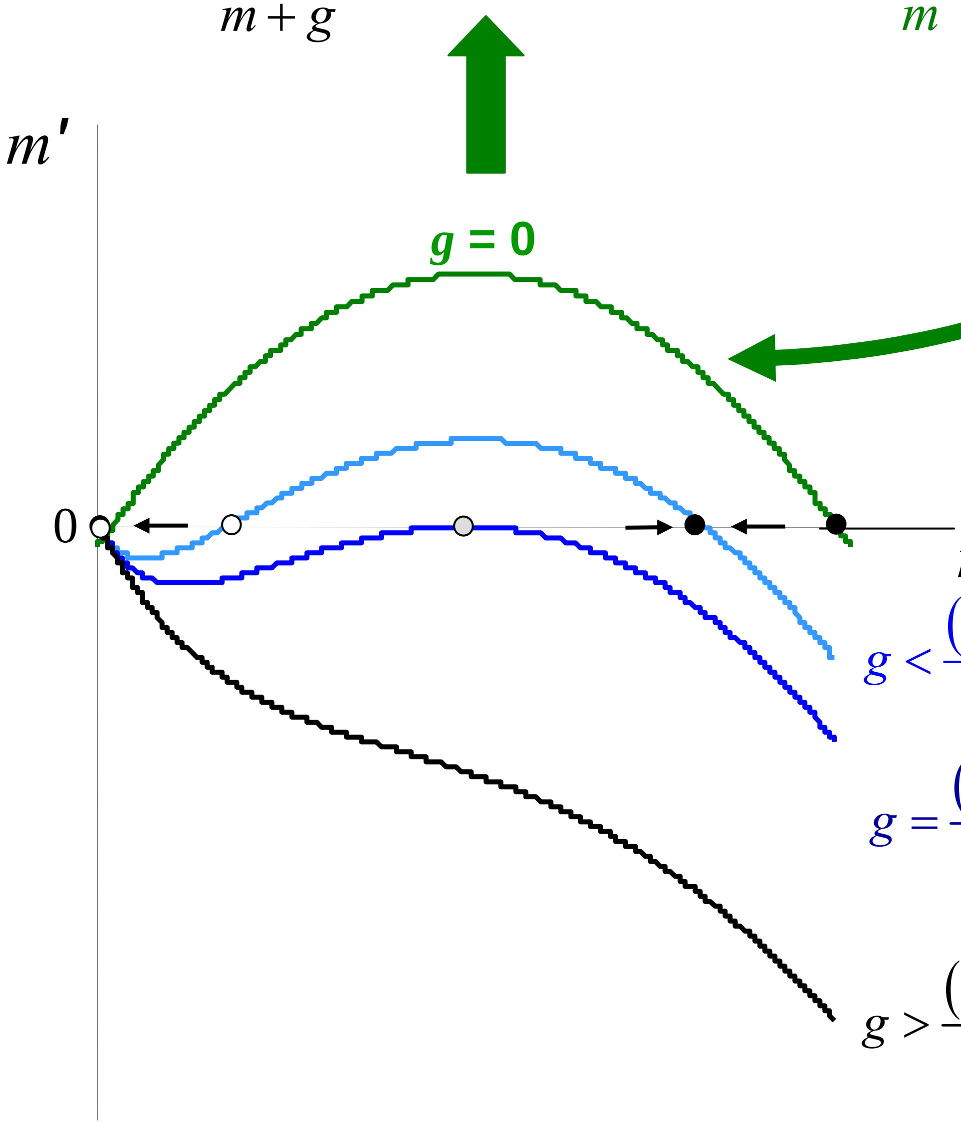
**Under √ :**

| | | |
|---|---|---|
| positive | $g < \frac{(R_0 - d)^2}{4cd} : m_2^*, m_3^*$ *exist and real* | **3 equilibria** |
| zero | $g = \frac{(R_0 - d)^2}{4cd} : m_2^* = m_3^*$ | **2 equilibria** |
| negative | $g > \frac{(R_0 - d)^2}{4cd} : m_2^*, m_3^*$ *do not exist (complex)* | **1 equilibrium** |

# Effect of $g$: multiple equilibria

$$m' = \frac{-cm^3 + (R_0 - d)m^2 - dg \cdot m}{m + g}$$

$$= \frac{-cm^3 + (R_0 - d)m^2}{m} = (R_0 - d)m - cm^2$$

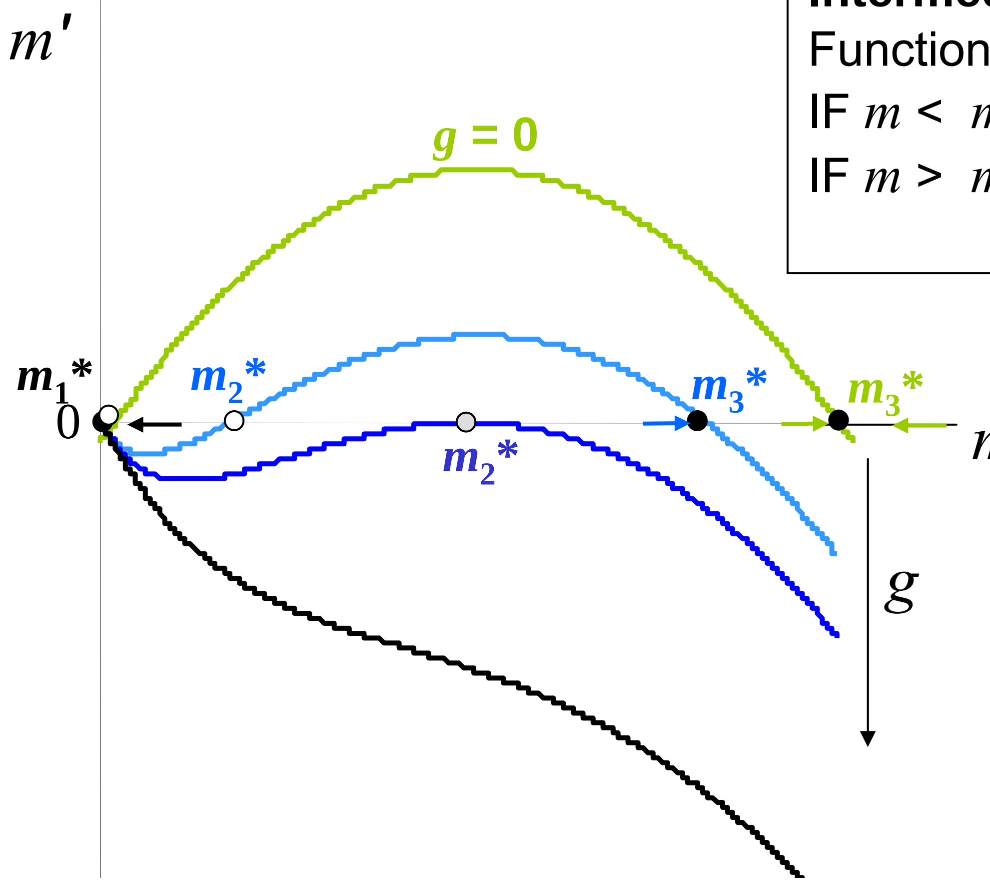


$g < \frac{(R_0 - d)^2}{4cd}$ **3 equilibria**

$g = \frac{(R_0 - d)^2}{4cd}$ **2 equilibria**

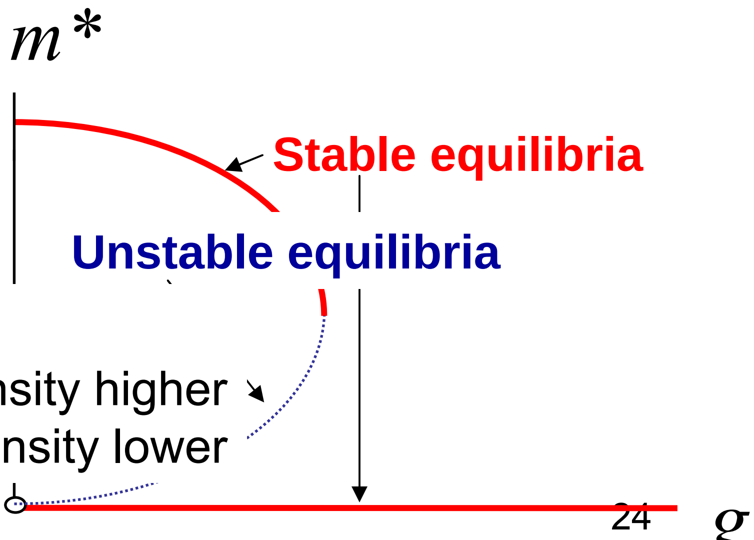
$g > \frac{(R_0 - d)^2}{4cd}$ **1 equilibrium**

# Effect of *g*: dynamic threshold



**Intermediate unstable fixed point ($m_2^*$)**
Functions as a ***set point:***
IF $m < m_2$ THEN $m$ goes extinct
IF $m > m_2$ THEN $m$ stabilizes at $m_3^*$

"catastrophe"



Bifurcation Diagram

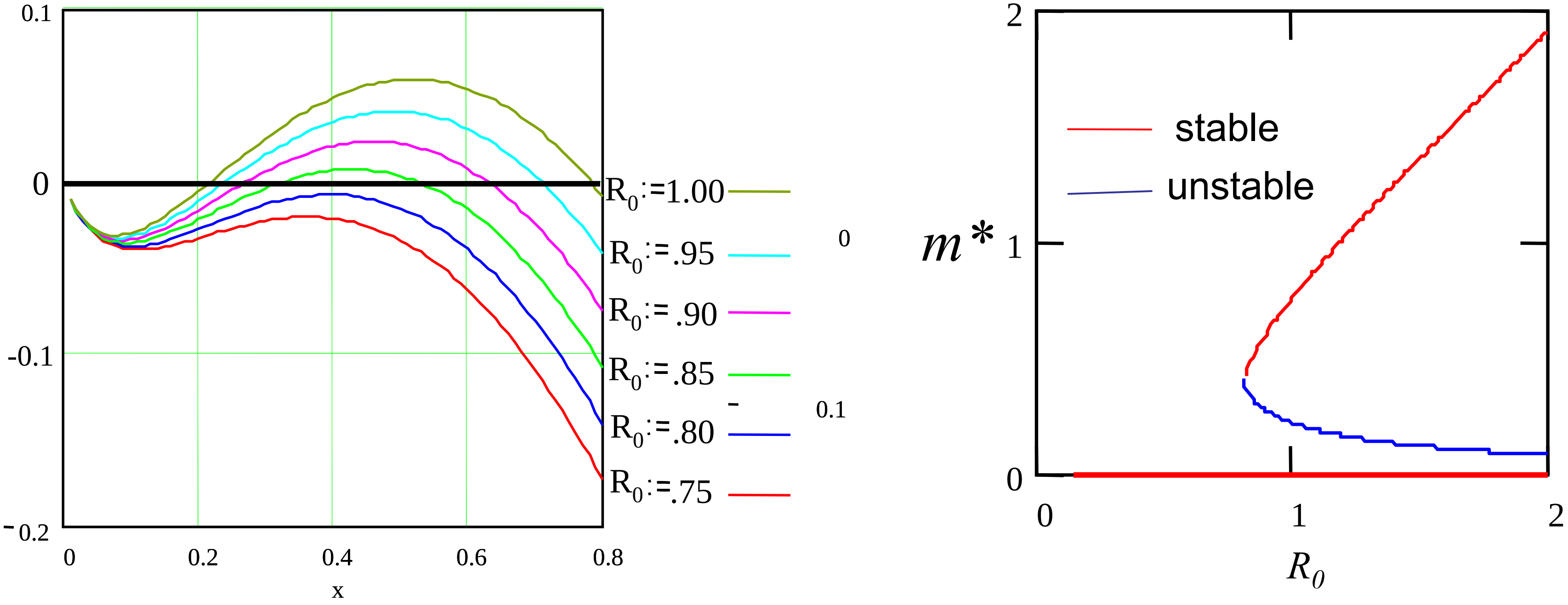
***Threshold is set by g (Deficit!)***
$g$ lower ⇒ $D$ higher ⇒ threshold lower, final intensity higher
$g$ higher ⇒ $D$ lower ⇒ threshold higher, final intensity lower

## Effect of $R_0$

Lower $R_0$:

moves unstable fp ($m_2$*) to the right (= higher threshold)

and stable fp (>0 = $m_3$*) to the left (= lower upper asymptote)



## **EXTENSION 3**: $\boldsymbol{R}$ as **Reward**

Carrying out the behaviour (expression of the tendency) is rewarding in itself ⇒ production of rewarding internal opiates:

$R$ = [endorphin]~$m_3^*$

For example, $R_{n+1} = p \cdot m_3^*$

If $0 < p < 1$, $R_{n+1} < R_n$

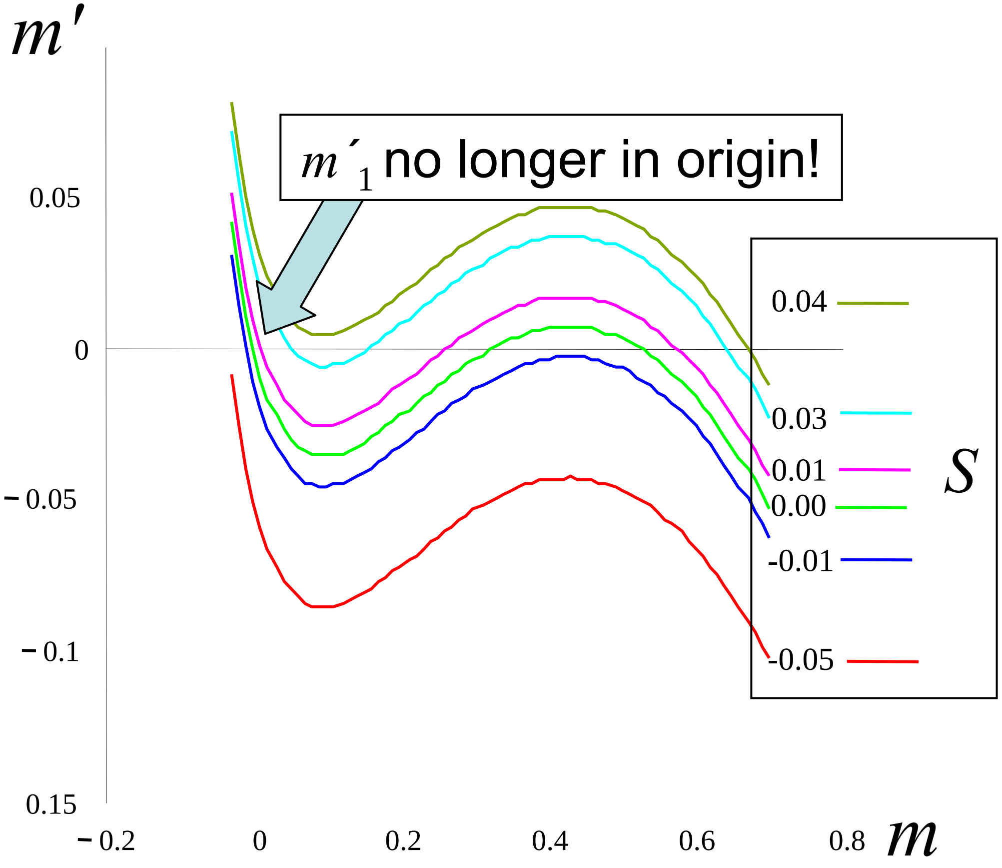
When $R$ gets smaller, $m_2^*$ moves up (and at the same time sets $m_3^*$ lower!) ⇒ next $R$ smaller than previous $R$

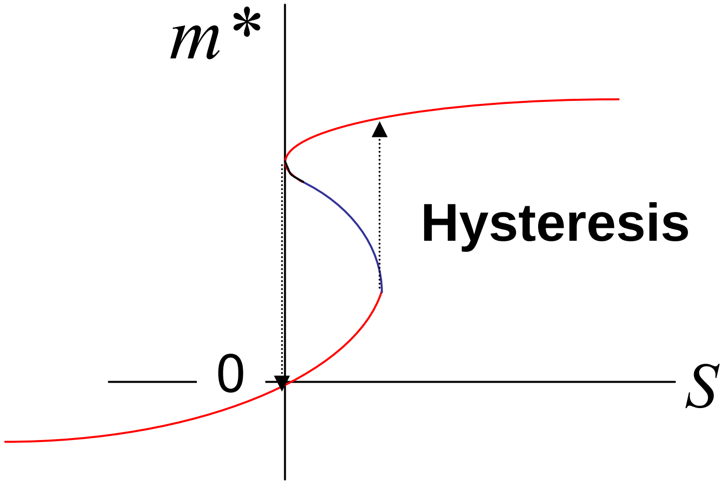
⬇

threshold higher, less “satisfaction”
= harder work for less pleasure!
(addiction?)

**EXTENSION 4**

# The effect of adding an external stimulus, $S$, of varying strength ("Cue Strength")





**Adding a constant S**
= shifting up the curve
= lower threshold,
higher final intensity

## EXTENSION 5

# Linking Stimulus intensity to Behavioural Tendency

$$m' = \frac{-am^3 + bm^2 - cgm}{m + g} + S$$

**Cue-strength as a function of behavioural tendency (and itself)?**

$$S' = \underset{\downarrow}{m} - \underset{\searrow}{S}$$

habituation

pos.feedback as in taste

Structurally very similar to Fitzhugh-Nagumo equation for neurotransmission

**Compare:** $x' = -mx^3 + bm^2 - cgm + S_0 - S$

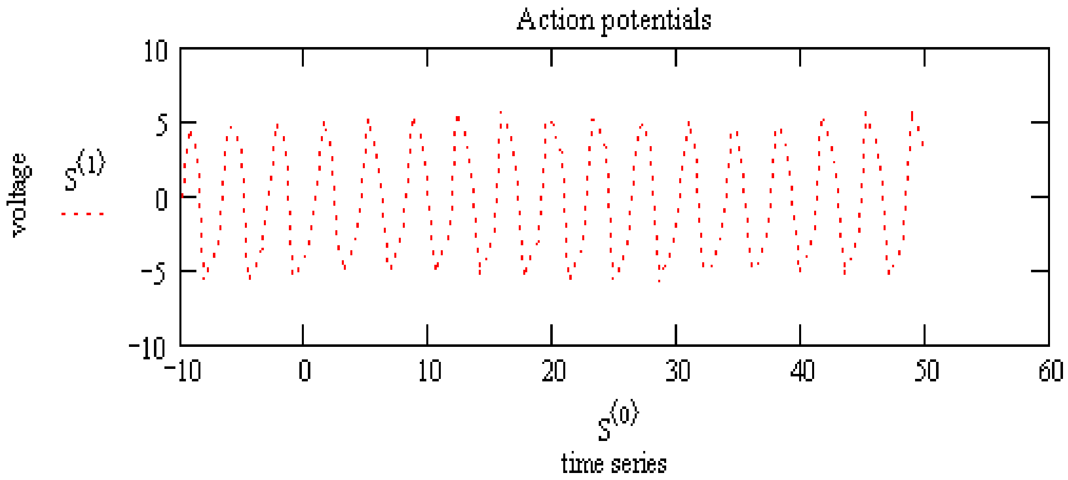
$S' = m - S$

- Oscillations, bursts, excitatory

# Fitzhugh-Nagumo Equations

- Relaxation oscillators

  Increase and decrease on different time scales



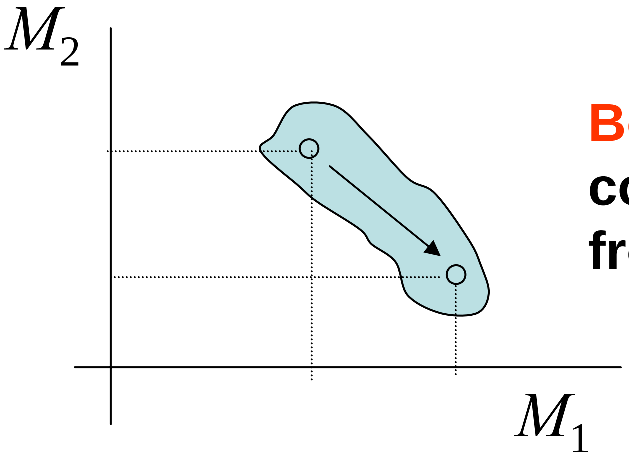
- Depending on parameters:

  oscillations, bursts, excitatory behaviour

phase-locking

# 2. Behaviour and Sensor-Motor Coordination

Ultimately: coordinated contraction of a group of muscles
(after stimulation from neurons, supported by
blood supply, carrying hormones, oxygen, etc.)



**Motor Space**

**Behaviour: region in “muscle state space” containing a trajectory that moves the system from one particular “muscle-state to another**

# Easier to comprehend:

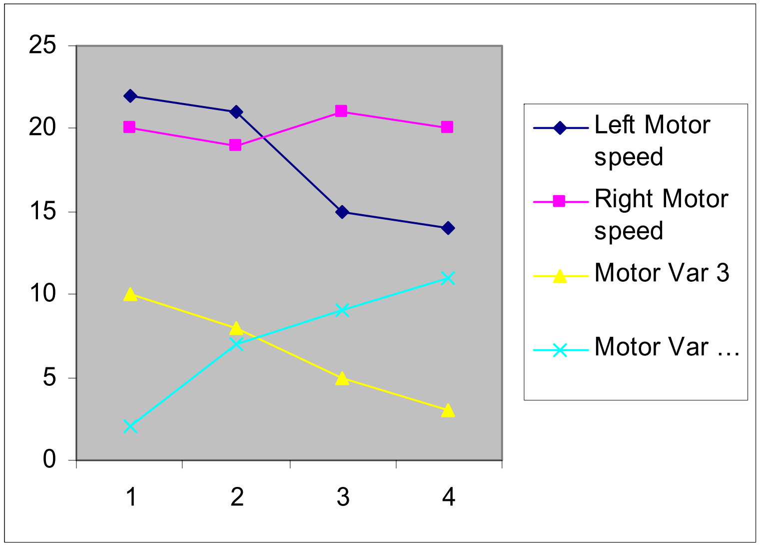
## Behaviour:

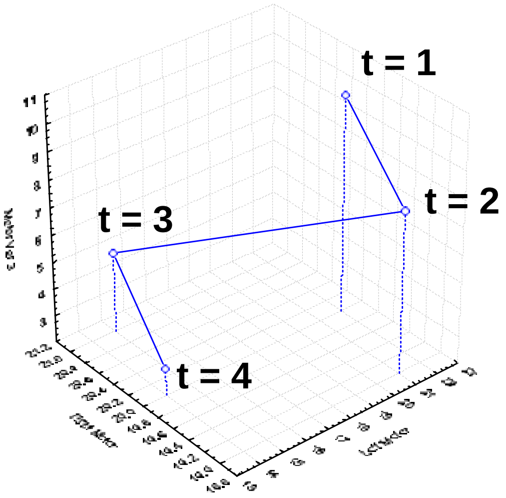
**Region in the "motor state space" of an *autonomous agent** *containing a trajectory that moves the system from one particular "motor-state" to another**

* e.g. a *robot*

# Measure the value of the actuators (motors) at every moment in time

| TIME | **Left Motor speed** | **Right Motor speed** | **Motor Var 3** | **Motor Var** |
|---|---|---|---|---|
| t =1 | 22 | 20 | 10 | 2 |
| 2 | 21 | 19 | 8 | 7 |
| 3 | 15 | 21 | 5 | 9 |
| 4 | 14 | 20 | 3 | 11 |
| etc | | | | |



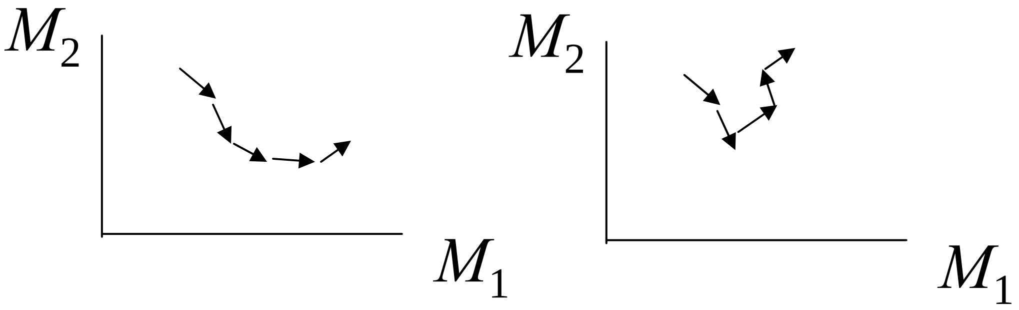


Coordinated action: correlations among motor variables over time

# If motor variables are correlated:

1.Shift in state space will be “coordinated", else “haphazard”

2. Dimension reduction



However, the degree of “correlatedness” ), and hence the number of dimensions, may shift in time (when system shifts from one behaviour to the other

# Sensor-Motor Coordination

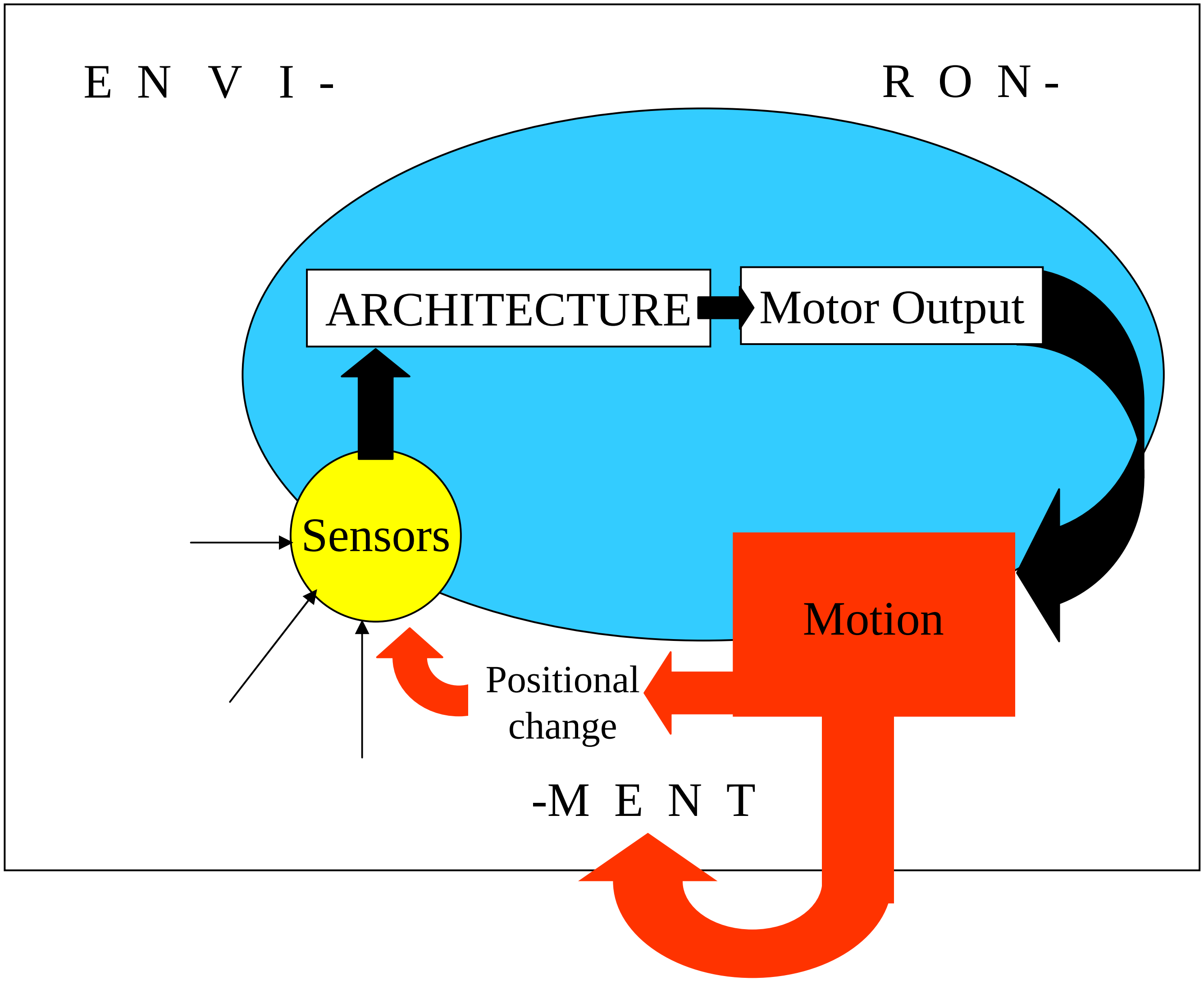
René te Boekhorst[1], Max Lungarella[2] & Rolf Pfeifer[3]

[1] Adaptive Systems Research Group, University of Hertfordshire, UK
[2] Neuroscience Research Institute,  Tsubuka, Japan
[3] **A.I. Lab, University of Zürich, Switzerland**

***(where the job was done)***

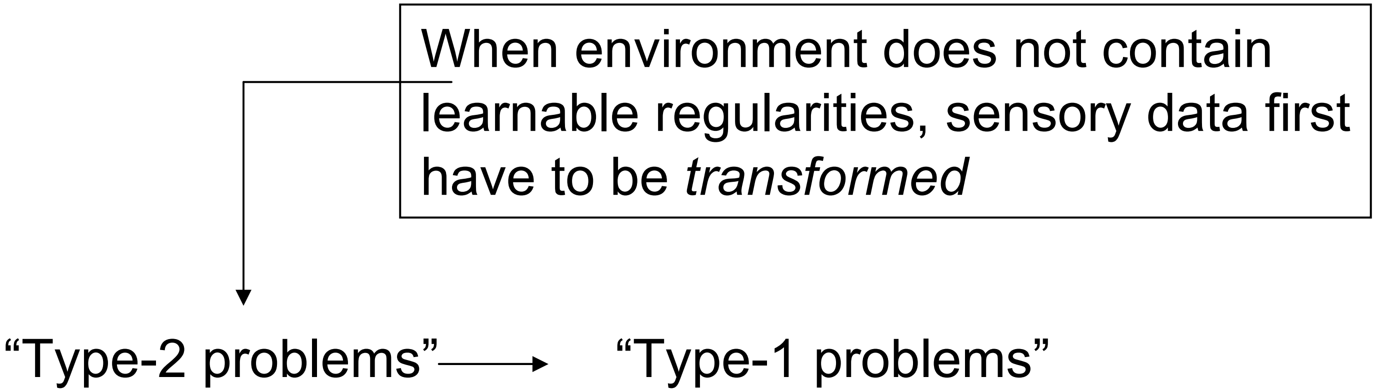
# ***Sensory Motor Control**:*

*The way an agent perceives its environment depends on what it is doing **and vice versa***



# **Starting Point**: Clarke & Thornton:

Behavioral & Brain Sciences (1997), 20:1



When environment does not contain learnable regularities, sensory data first have to be *transformed*

“Type-2 problems” → “Type-1 problems”

## **Thesis**: Pfeifer & Scheier:

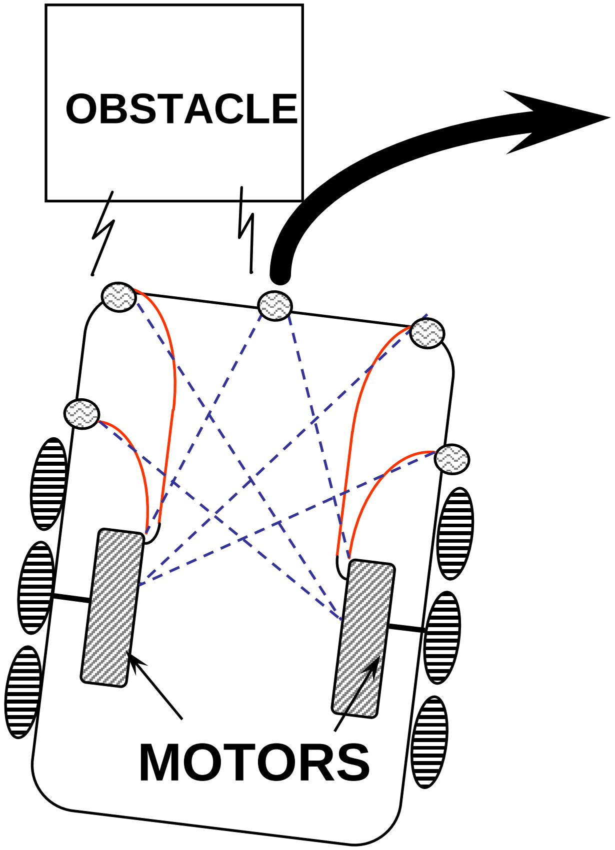
Zeitschrift für Naturforschung (1998), 53c, 480-503

***Sensory Motor Control*** provides means for this transformation

# Object Avoidance by a Braitenberg Vehicle

**(Braitenberg, 1984)**

**Architecture Didabot**



**Excitatory Connection**

**Obstacle Detector (IR sensor)**

**Inhibitory Connection**

# Example:

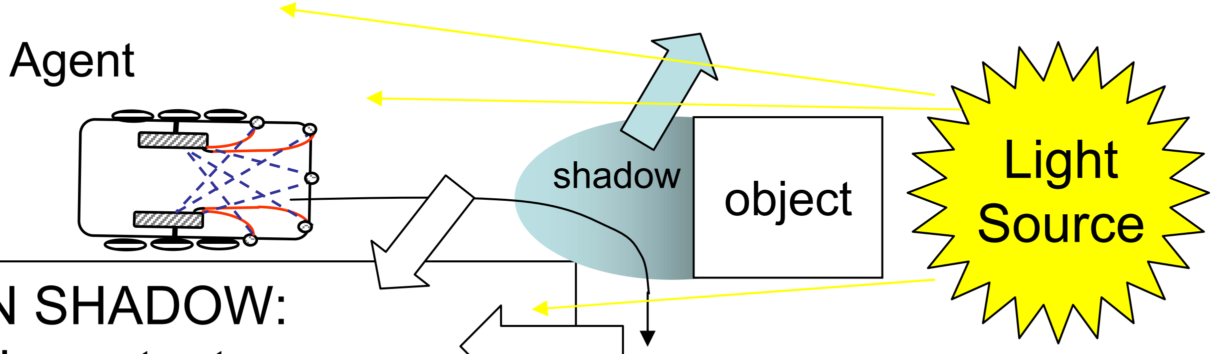
Agent attracted to light (measured by Ambient Light Sensors)

↓

Avoid shadows, and hence obstacles
(distance to objects measured by Infra Red Sensors)

IN SHADOW :

- Increasingly **lower values** for Amb. Light Detector and **higher output** of IR sensors
- Left motor (wheel) spins faster than right motor (wheel)



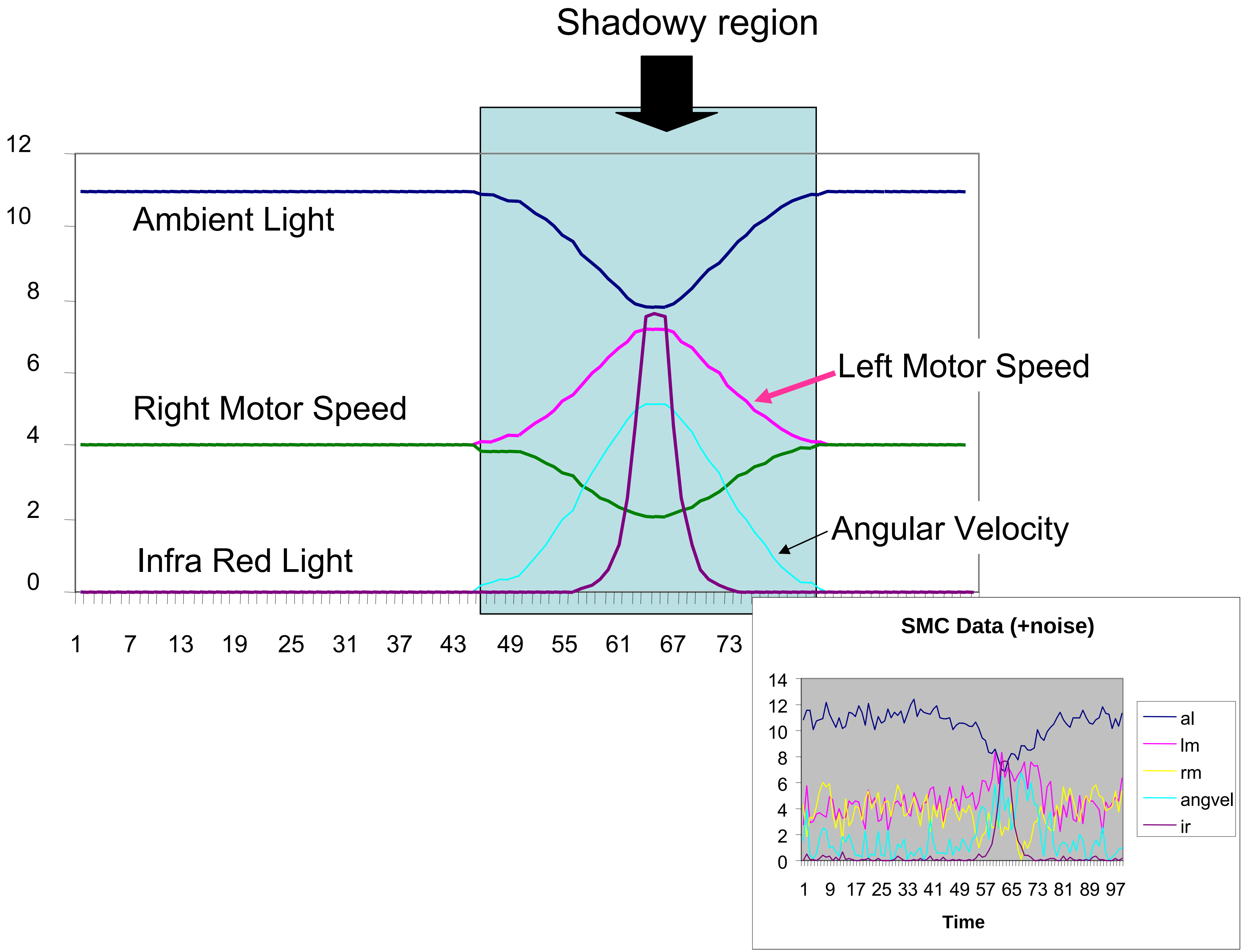
NOT IN SHADOW:

•Speed constant;

•Amb.Light detection: Maximal

•Infra Red Sensors: No output

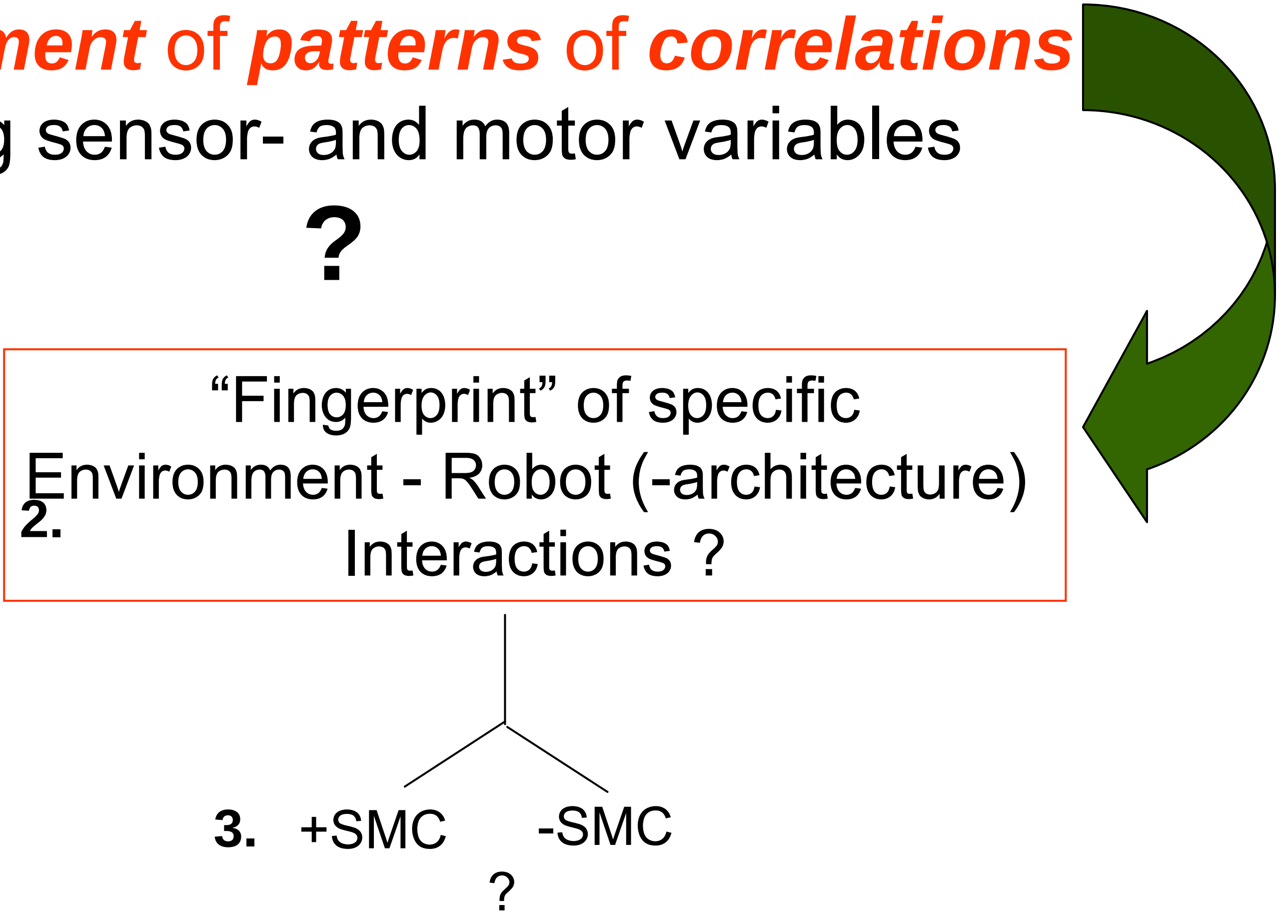
***Sensory Motor Control leads to correlations among sensory motor channels***


Shadowy region
Ambient Light
Right Motor Speed
Left Motor Speed
Angular Velocity
Infra Red Light
12
10
8
6
4
2
0
1
7
13
19
25
31
37
43
49
55
61
67
73
SMC Data (+noise)
14
12
10
8
6
4
2
0
1 9 17 25 33 41 49 57 65 73 81 89 97
Time
al
lm
rm
angvel
ir

# Research Questions

**1.** How to identify the possible
***Development*** of ***patterns*** of ***correlations***
among sensor- and motor variables

**?**

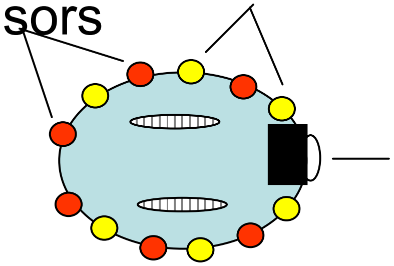


**2.** “Fingerprint” of specific
Environment - Robot (-architecture)
Interactions ?

**3.** +SMC -SMC
?

# The Agent

“Samurai” Robot



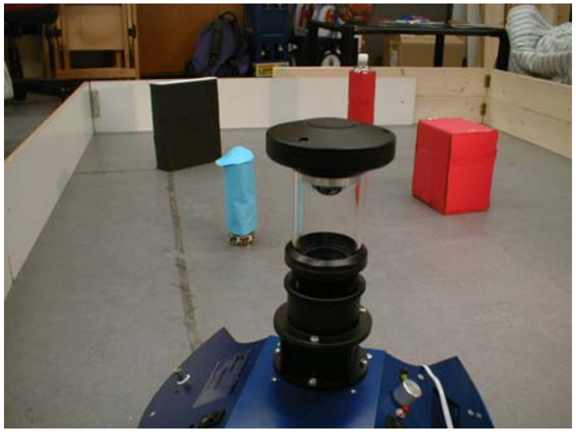
12 IR Sensors

12 AL Sensors

Camera

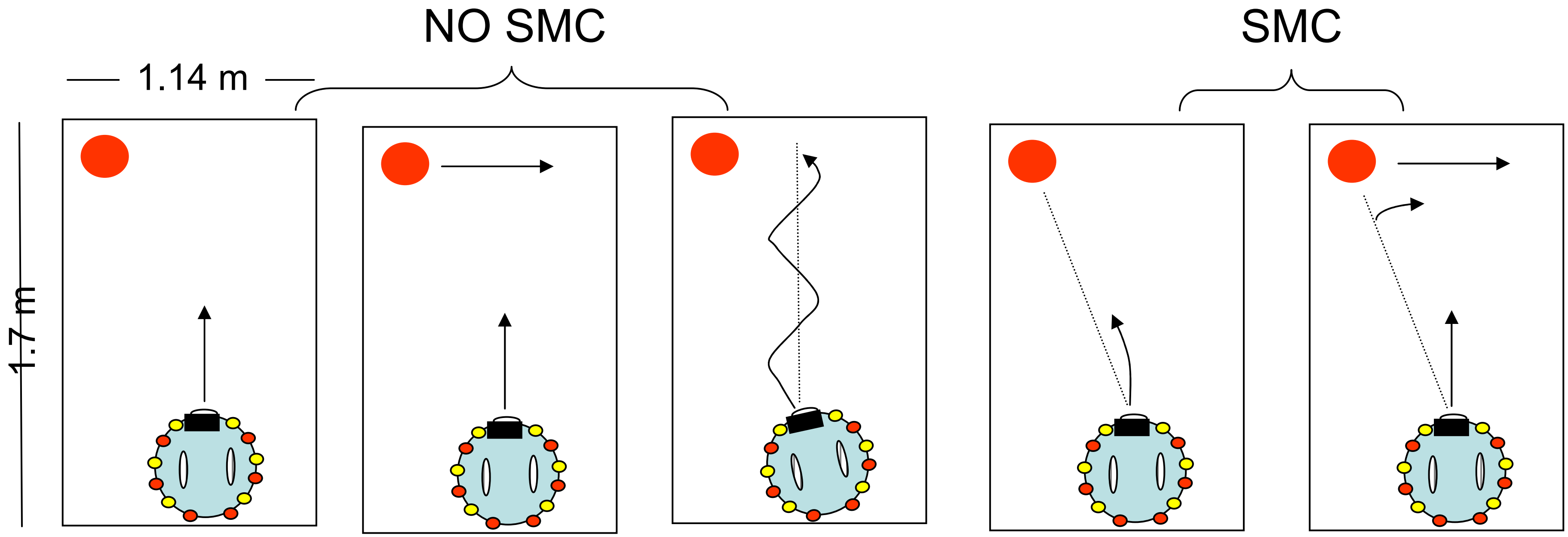
Image = strip of 100 pixels

# Experimental Set-Up



Each Experiment:

- Every 2-3 sec data transmitted to computer (100 time steps)
- 15 replications



Moving object

Meandering Robot

Object focusing

Track moving object

# METHODS

## Typical Data Set

| | **Motor OutPut** | | | **Sensor Input** | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | Ang.Velocity | **Infra Red Sensors** | | | | **Ambient Light Sensors** | | | | **Image Data** (pixel locations) | | | |
| Time | Right Motor (RM) | Left Motor (LM) | (=RM-LM) | IR-Sensor 1 | IR-Sensor 2 | … | IR-Sensor $n_i$ | AL-Sensor 1 | AL-Sensor 2 | … | AL-Sensor $n_a$ | 1 | 2 | … | 100 |
| 1 | $RM_1$ | $LM_1$ | $AV_1$ | $IR1_1$ | $IR2_1$ | … | $IRn_{i1}$ | $AL1_1$ | $AL2_1$ | … | $ALn_{a1}$ | $I1_1$ | $I2_1$ | … | $I100_1$ |
| 2 | $RM_2$ | $LM_2$ | $AV_2$ | $IR1_2$ | $IR2_2$ | … | $IRn_{i2}$ | $AL1_2$ | $AL2_2$ | … | $ALn_{a2}$ | $I1_2$ | $I2_2$ | … | $I100_2$ |
| 3 | $RM_3$ | $LM_3$ | $AV_3$ | $IR1_3$ | $IR2_3$ | … | $IRn_{i3}$ | $AL1_3$ | $AL2_3$ | … | $ALn_{a3}$ | $I1_3$ | $I2_3$ | … | $I100_3$ |
| . | . | . | . | . | . | … | . | . | . | … | . | . | . | … | . |
| . | . | . | . | . | . | … | . | . | . | … | . | . | . | … | . |
| . | . | . | . | . | . | … | . | . | . | … | . | . | . | … | . |
| N | $RM_N$ | $LM_N$ | $AV_N$ | $IR1_N$ | $IR2_N$ | … | $IRn_{iN}$ | $AL1_N$ | $AL2_N$ | … | $ALn_{aN}$ | $I1_N$ | $I2_N$ | … | $I100_N$ |

Correlating all variables with each other ?

Problems:

- (Very) large number of variables → Bonferonni inflation
- Some sets contain many more variables than others (e.g. image data)
- Correlations may change over time

# Proposed Methodology:

1. Dimension Reduction (PCA / Factor Analysis)

   for each sensor modality (AL, IR, Images)

2. Correlate the PCs

3. Summarize the correlations

   (e.g. the determinant of the correlation matrix)

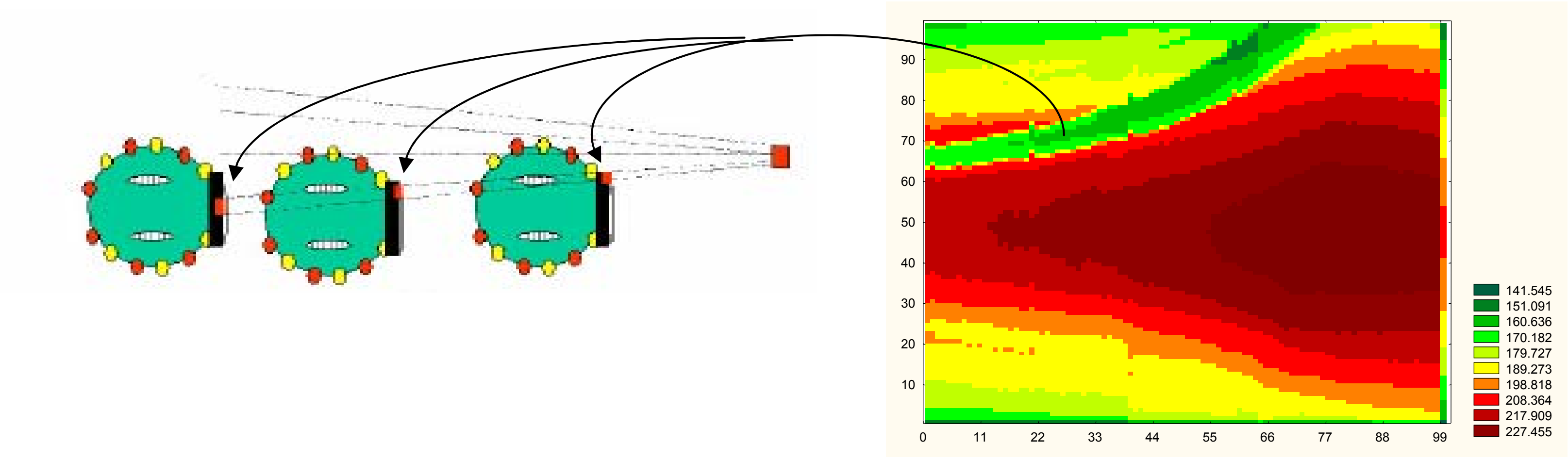
$$\begin{array}{c|c|c|}
 & X & Y \\
\hline
X & r_{xx} = 1.00 & r_{xy} \\
\hline
Y & r_{yx} & r_{yy} = 1.00 \\
\hline
\end{array}$$

$$|\mathbf{R}| = 1 - r^2_{xy}$$

$1 - |\mathbf{R}|$ = prop. of variance of X due to variance of Y

4. Calculate $|\mathbf{R}|$ at various points in time (e.g. for a moving window)

# How the robot sees the world



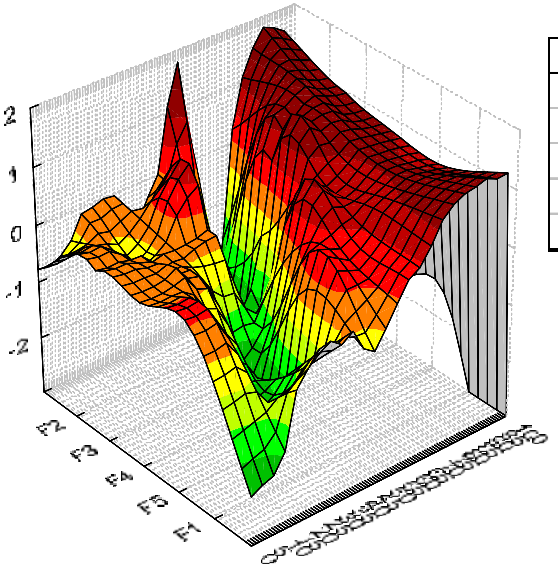
# Dimension Reduction

## Image Data

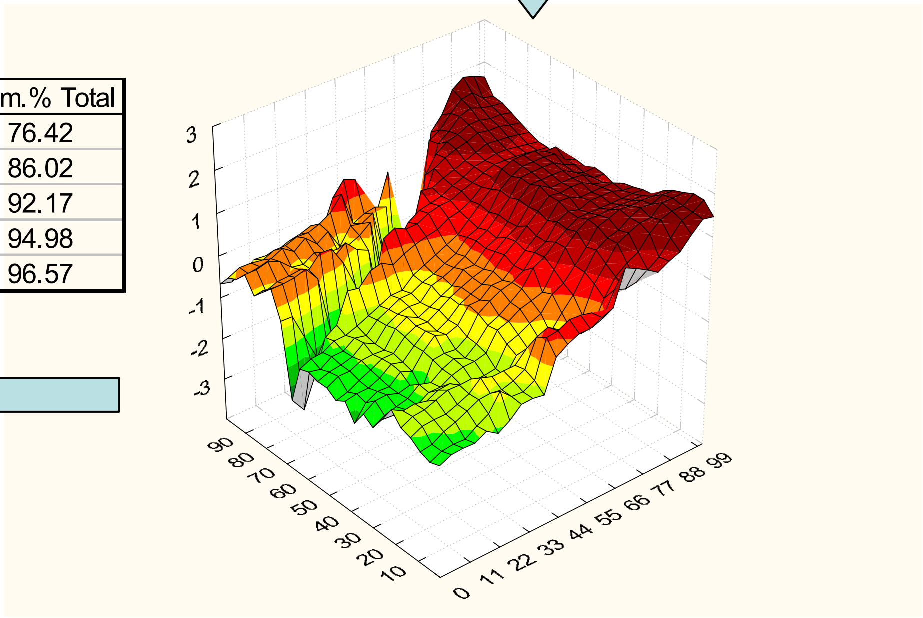
3D representation

PCA

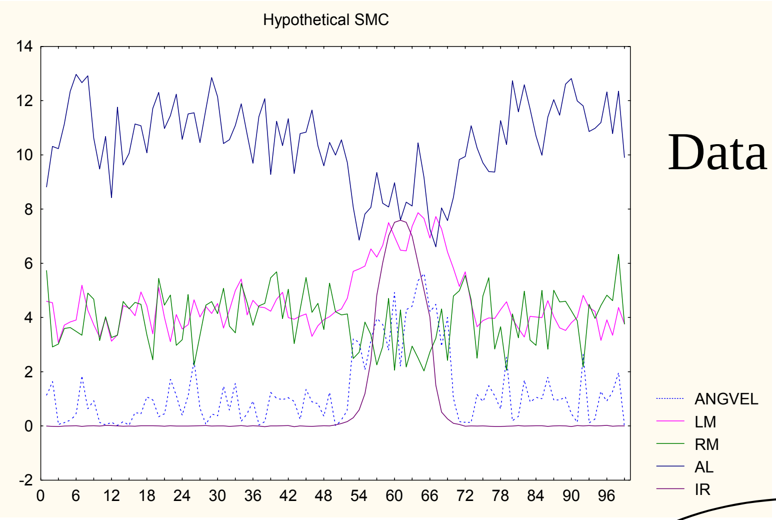
| | Eigenvalue | Cum.% Total |
|---|---|---|
| 1 | 75.66 | 76.42 |
| 2 | 9.58 | 86.02 |
| 3 | 6.01 | 92.17 |
| 4 | 2.78 | 94.98 |
| 5 | 1.58 | 96.57 |



Reconstruction based on 5 Factors



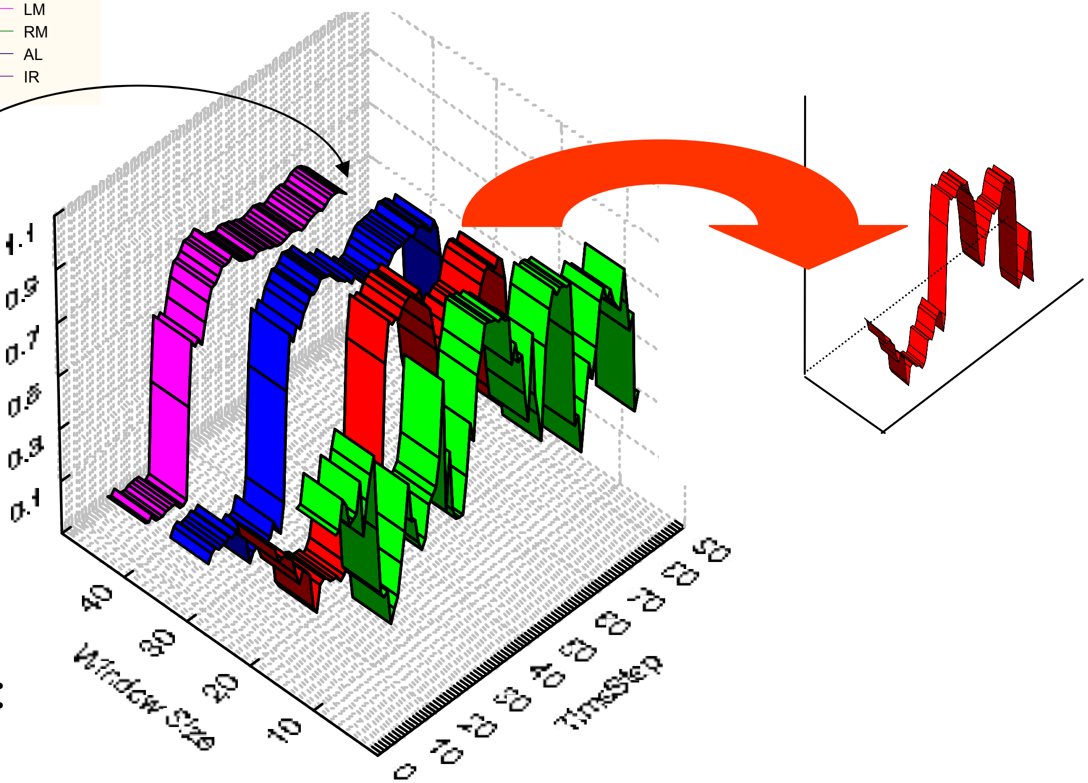
# Calculating $|\mathbf{R}|$ for various Moving Window sizes (W)



Data → Correlation Matrix

$1 - |\mathbf{R}|$

Problem of Moving Window: truncation at (N-W-1) data points

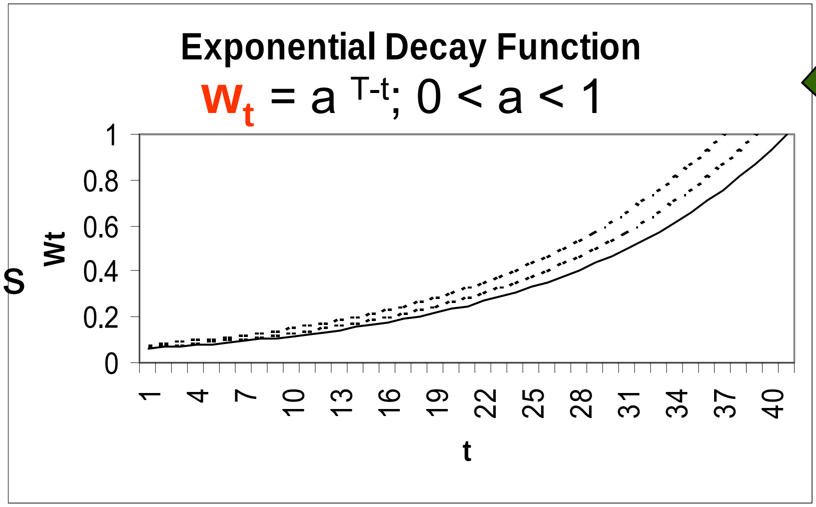


“Continuous” Alternative:
***Weighted Correlations***

# “Weighted” Correlations

$r_t$ calculated over 4,5, … , all (N) time steps but with ***diminishing influence of the past***

Decay function $w_t$



**Exponential Decay Function**

$w_t = a^{T-t}; 0 < a < 1$

a = 1: smoothed value = mean of time series

a = 0: smoothed value = last value

Use in Regression:

Every ***contribution*** to regression is weighted by the decay function:

$$r^{*}_{T} = \frac{\sum w_t x_t y_t - \sum w_t x_t \sum w_t y_t / N}{\left[\sum w_t x_t^2 - \left(\sum w_t x_t\right)^2 / N\right]\left[\sum w_t y_t^2 - \left(\sum w_t y_t\right)^2 / N\right]}$$

# Type of Decay Functions

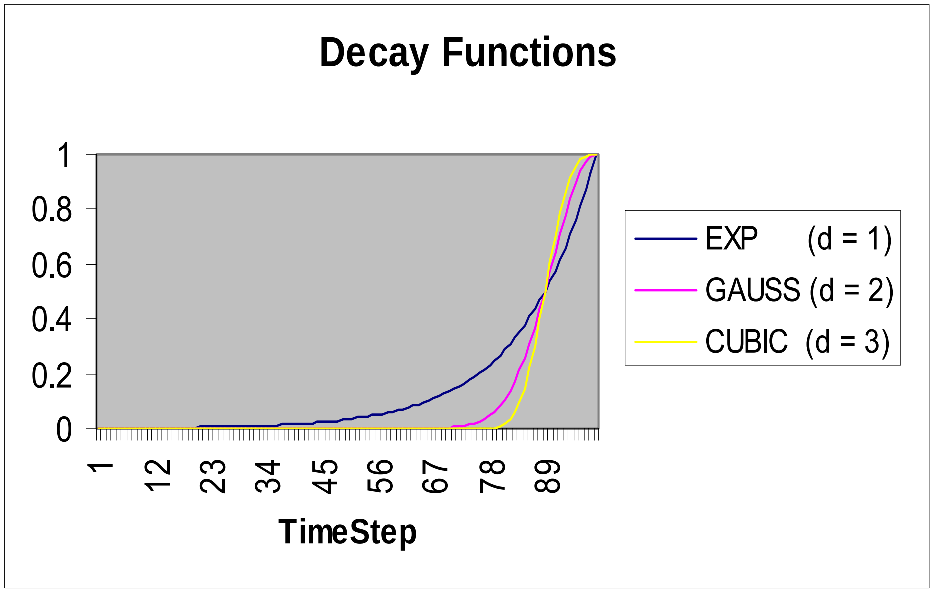
$$w_t = a^{T-t} = e^{(T-t)\cdot\ln(a)} \qquad 0 < a < 1 \quad \text{Exponential decay}$$

$$\text{General}: \; w_t = e^{(T-t)^d/cB} \quad B = \frac{1}{\ln(a)} \quad c = \text{constant}$$

$d = 1$ : exponential decay

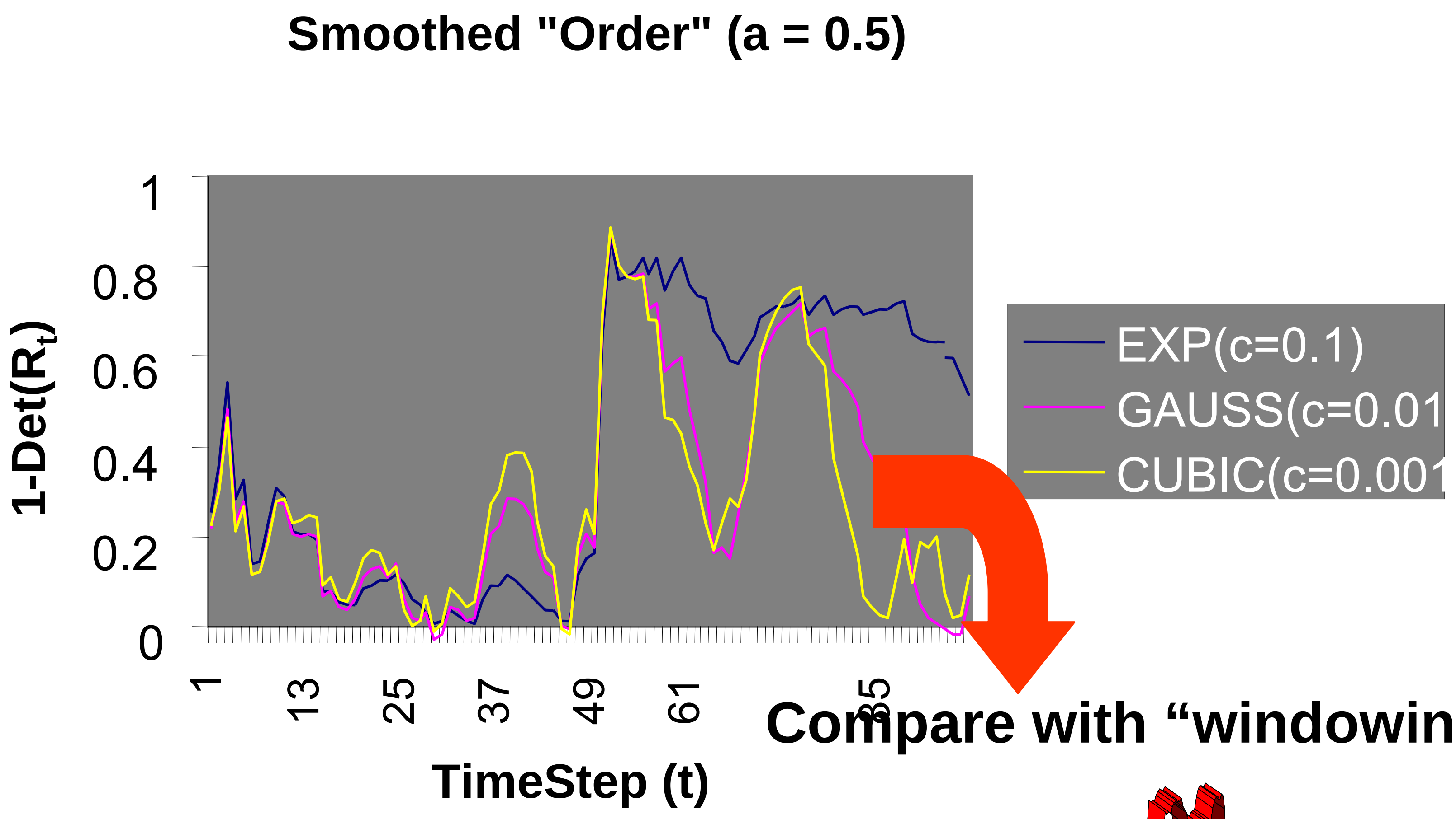
$d = 2$ : gaussian decay

$d = 3$ : cubic decay

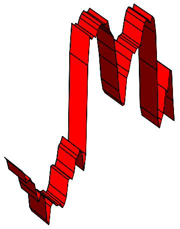


**Higher order decay function:**
Immediate past is preserved better,
Distant past is “forgotten” faster

# Application: Hypothetical SMC data



**Compare with “windowing”!**



# Robustness of 1 - $|\mathbf{R}|$

$|\mathbf{R}|$ approaches zero (or 1 - $|\mathbf{R}| \to 1$) for:

* Large number of variables
* Small value of smoothing coefficient *a*
= high “forgetting” rate (i.e. correlating over short ranges)

**Other characteristics of correlation matrices should be used**

# Characteristic Roots (Eigenvalues, $\mathbf{l}_i$)

$\lambda_i$ = % of variance explained by i-th principal component

BUT: as many eigenvalues as there are variables !

***How to combine them in a single characteristic number ?***

a. “Diversity” = Shannon Entropy:

$$H = -\Sigma \lambda_i \ln (\lambda_i)$$

At maximum when all $\lambda_i$ are identical

→ variance evenly distributed over PC’s

b. “Dominance” = Berger-Parker Index:

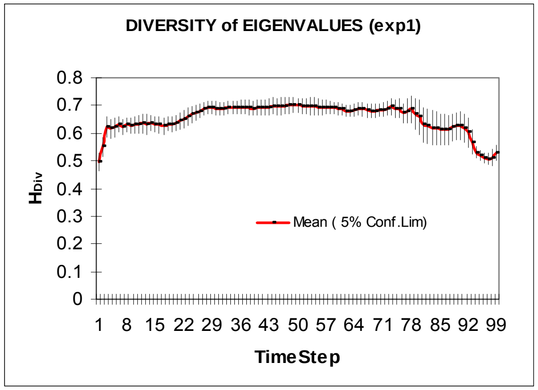
$$D = \lambda_{max} / \Sigma \lambda_i = \lambda_1$$

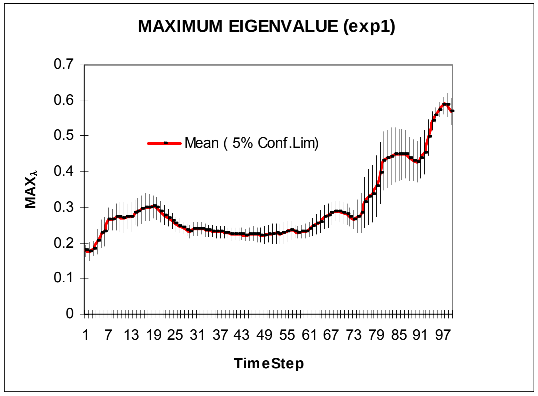
c. Number of Factors:

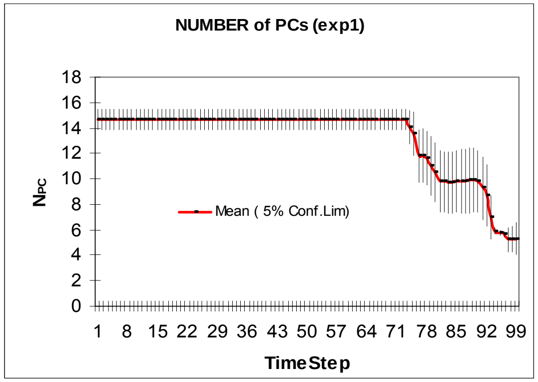
k = = number of PC´s that together explain

100$\alpha$ % of the total variance $\sum_j \lambda_j \geq \alpha$

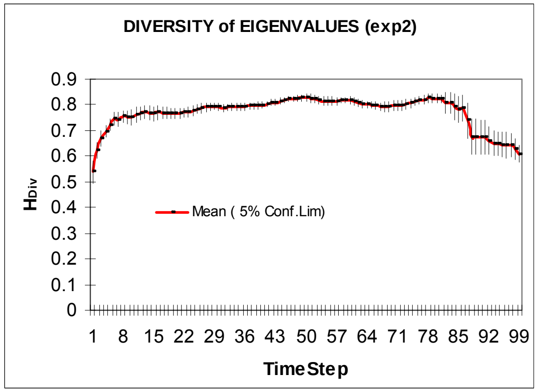
# EXP1: Control

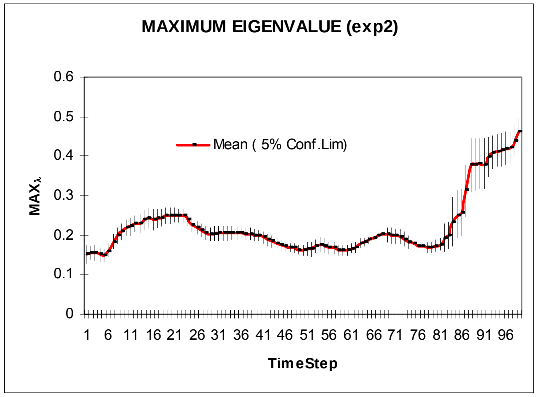

DIVERSITY of EIGENVALUES (exp1)

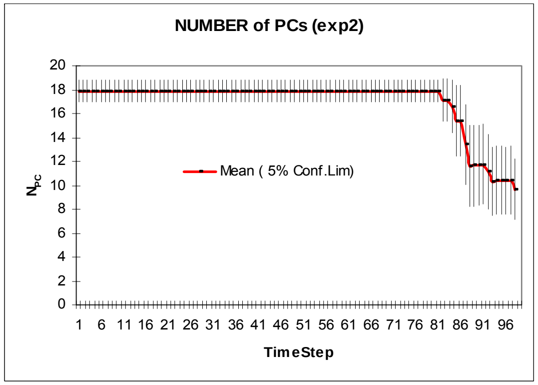

MAXIMUM EIGENVALUE (exp1)


NUMBER of PCs (exp1)

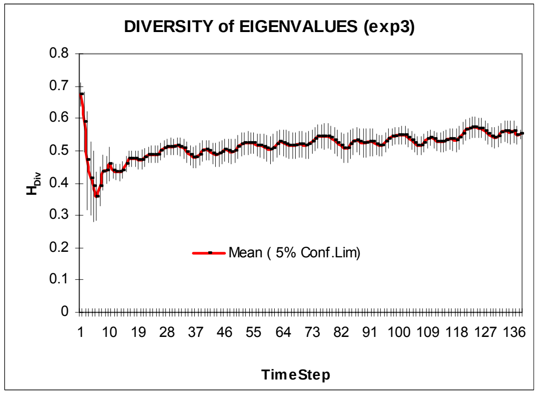
# EXP2: Moving object

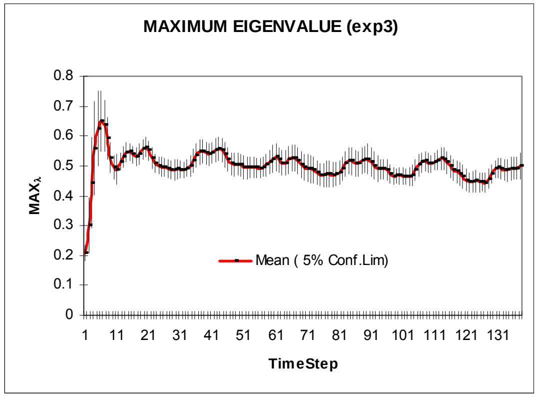

DIVERSITY of EIGENVALUES (exp2)

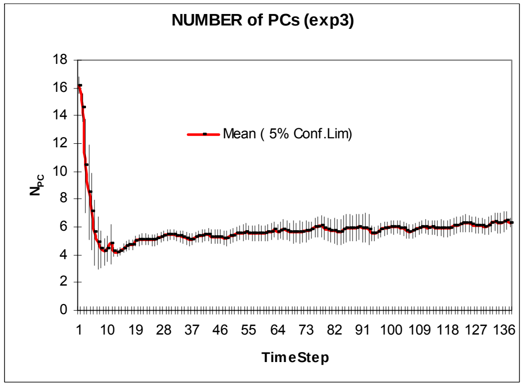

MAXIMUM EIGENVALUE (exp2)


NUMBER of PCs (exp2)

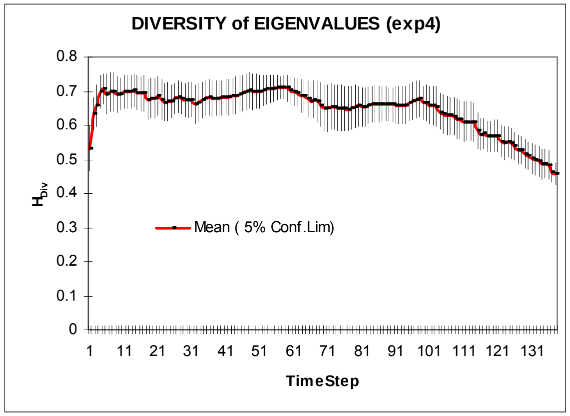
# EXP3: Oscillating

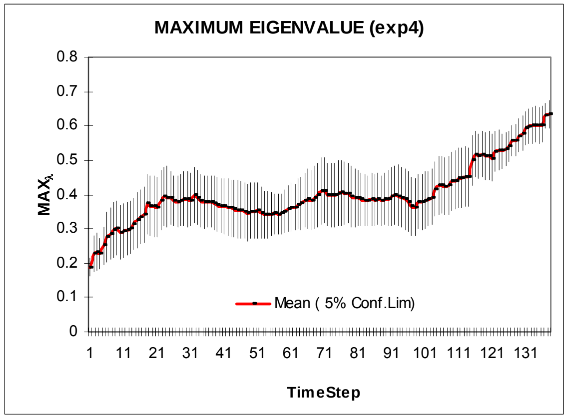

DIVERSITY of EIGENVALUES (exp3)

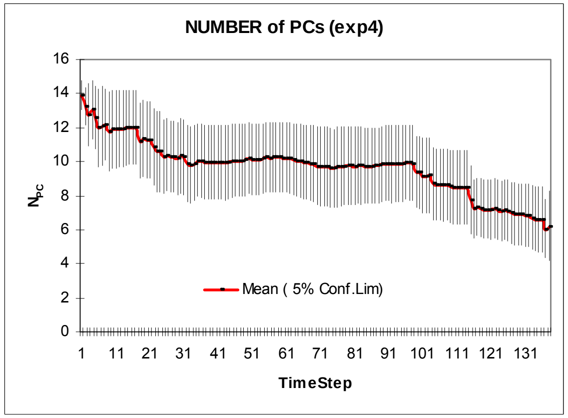

MAXIMUM EIGENVALUE (exp3)


NUMBER of PCs (exp3)

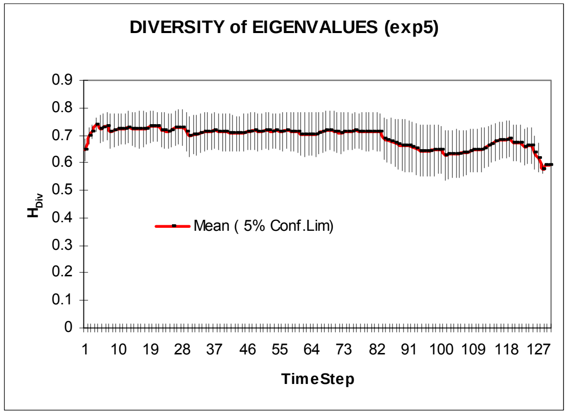
# EXP4: Simple tracking

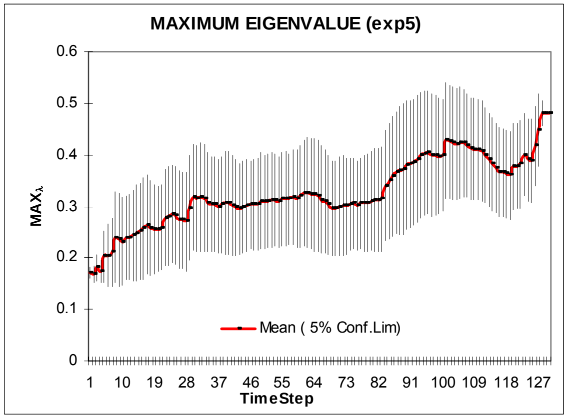

DIVERSITY of EIGENVALUES (exp4)

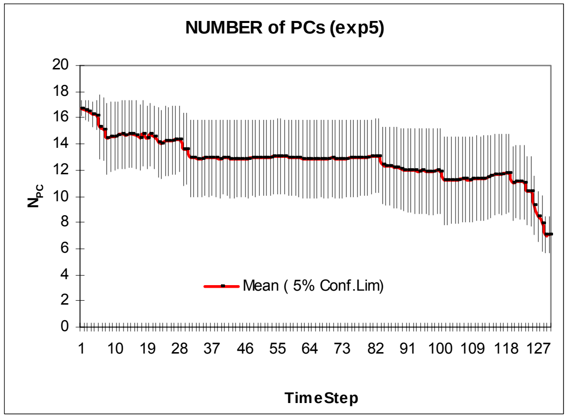

MAXIMUM EIGENVALUE (exp4)


NUMBER of PCs (exp4)

# EXP5:Tracking a moving object


DIVERSITY of EIGENVALUES (exp5)


MAXIMUM EIGENVALUE (exp5)


NUMBER of PCs (exp5)

# Other way of estimating association among variables:

- Mutual Information
- Information Distance

**= d(X,Y) = H(X|Y) + H(Y|X)**

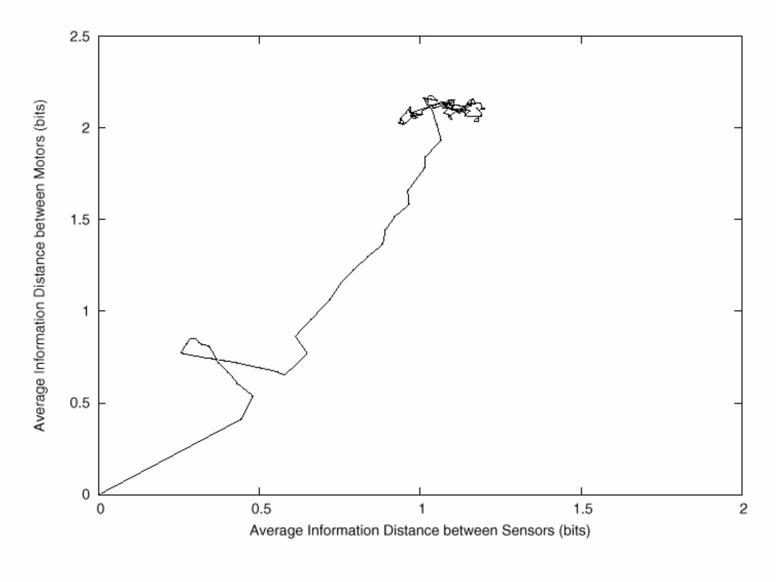
Entropy of variable X given
Entropy of variable Y

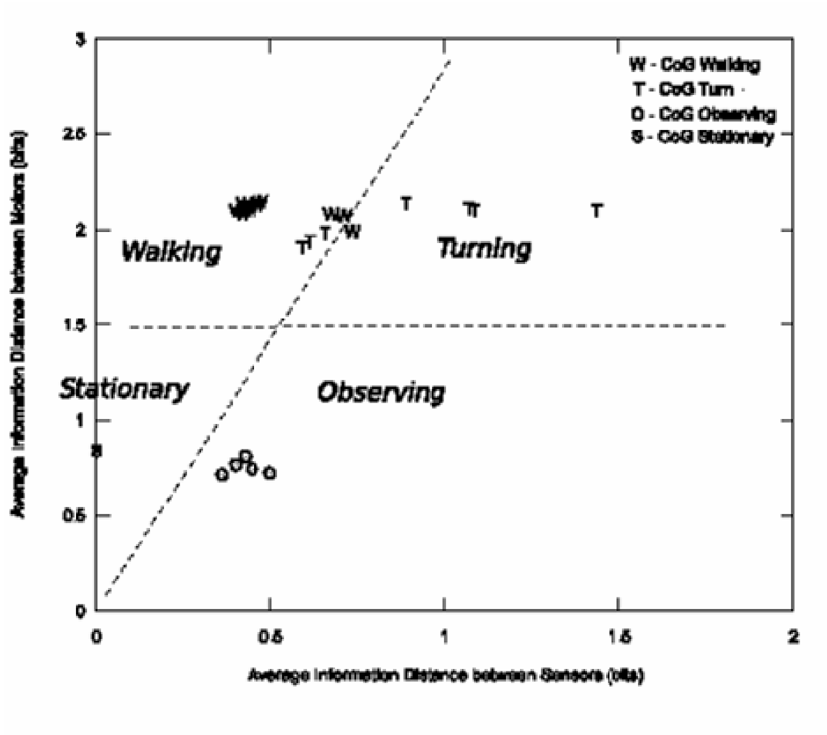
**Based on Shannon's Information Theory**

# *Interaction Histories and Action Selection*

(PhD project of Assif Mirza,
as part of the RobotCub Integrated Project)

- Sensor-Motor values (SMV) as points in a n-D space
- Trace change over time
- Shape/position of the trace represents expression of a particular act of behaviour





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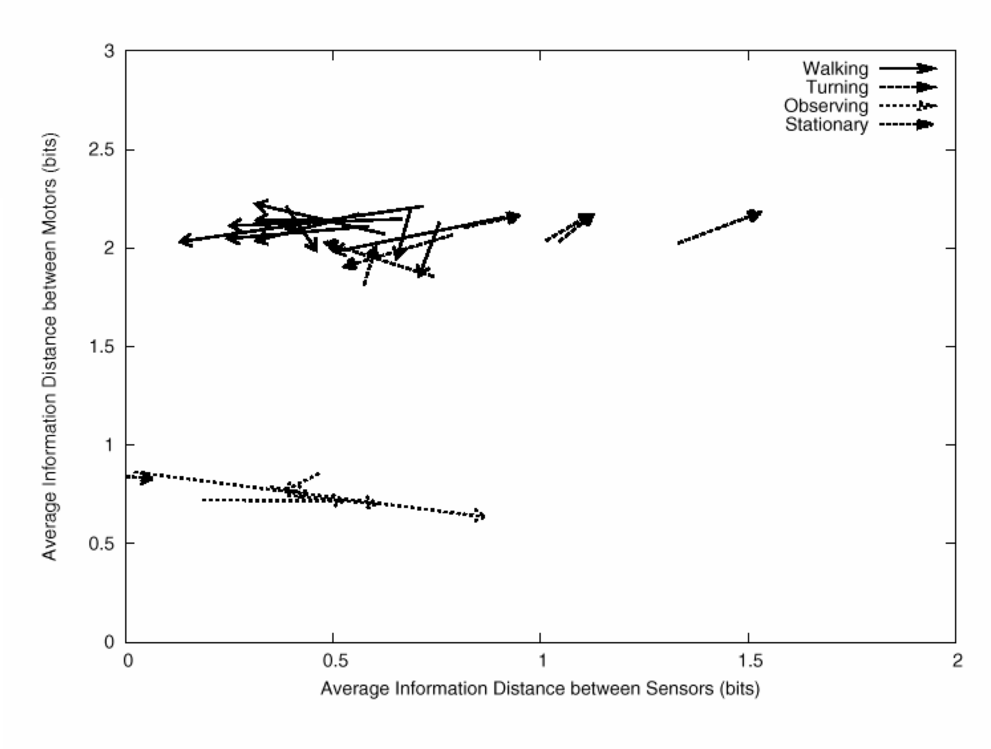


# 3. Dynamics Driving Behaviour

*What drives* ***motion*** *of a point in Sensor-Motor State Space ?*

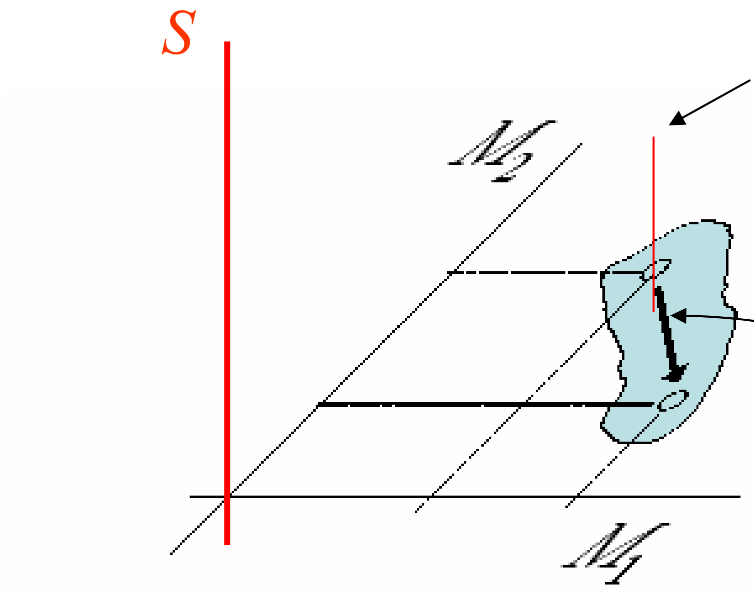
↓

“**Motivation**” = Dynamics of Behaviour Tendencies

To each point in Motor space is associated
the dynamics of $m'(t) = f(m, Ro, g, S$ etcetera)

“Attachment” of $m'(t)$ at a point $M_1$, $M_2$, $M_3$, ..
is realized via the dimension S in the sensor-motor space

# Behaviour: Motion in SM State Space driven by Behavioural Tendency



Sensory input:

$$m' = \frac{-am^3 + bm^2 - cgm}{m + g} + S$$

Motor value,
contributing to ***deficits***

Together with:
- other deficit components
- $m1', m2'$ , …

leads to $m_3^*$ (or $m_1^*$)

$M_i' = f(M_i, m_3^*)$
for example: $m_j^*\ f(M_i)$

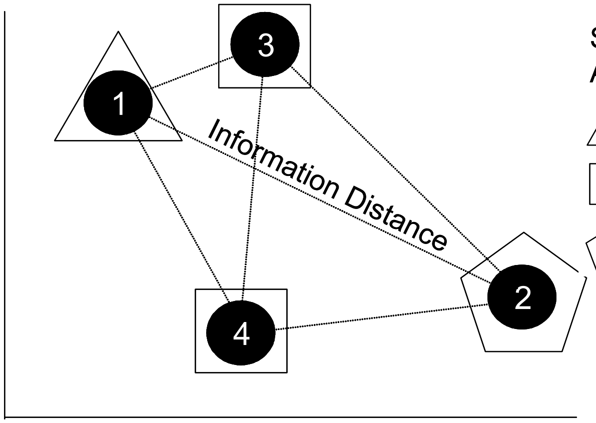
If $m_j^* = 0 \Rightarrow$ no change in M
Else: proportional to behavioural tendency

# “PEEK-A-BOO” with AIBO

RobotCub.org

A “quick-and-dirty hack” to get all this in a robot

- Sensor-Motor values (SMV) as points in a n-D space
- Current SMV is compared with previous ones
- A value (quality) is attached to an SMV, making it an “*experience*”
- Next Action = Action following nearest SMV neighbour with highest quality

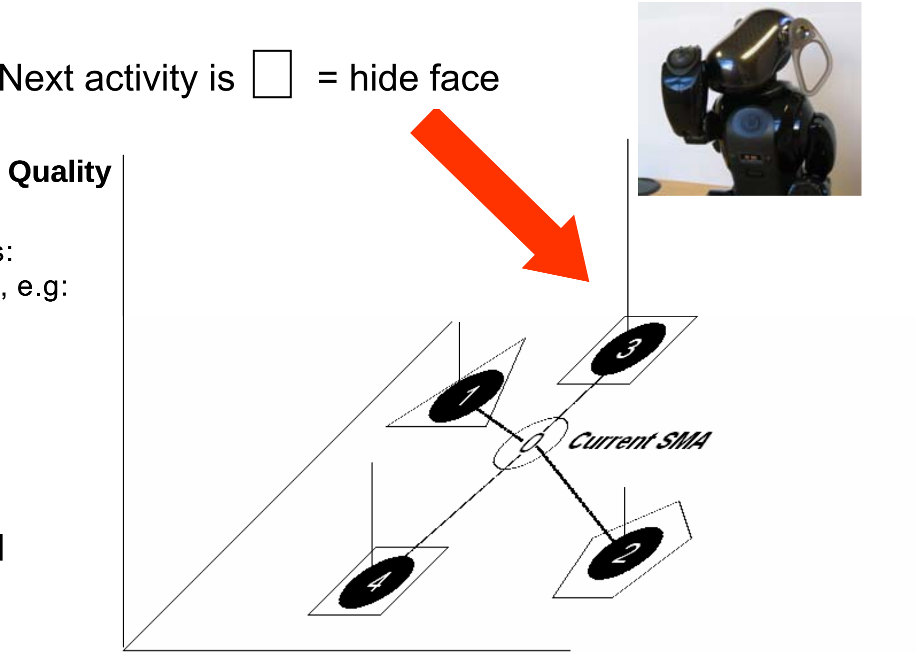


Symbols surrounding points:
Activity subsequent to SMA, e.g:

△ = raise front paws
☐ = hide face
⬠ = turn head

Numbers correspond to order in time

Next activity is ☐ = hide face



Information Distance = d(X,Y) = H(X|Y) + H(Y|X)

# Quality of SMA = Intensity of "Attention"

**Determined by internal dynamics + external stimulus (= S)**

"Attention" is fuelled by "Expectation" (= "resource"),

"Expectation" decreases at the expense of "Attention"

**Attention** increases on seeing a face (**S = 1**), but decreases on "loosing" a face (**S = 0**)

$$x_{t+1} = x_t + [(\overbrace{r_0 - cx_t}^{r_t = \text{Expectation}})x_t]S_t - dx_t(1-S_t)$$

if $S_t = 1$: $x_{t+1} = x_t + (r_0 - cx_t)x_t$
if $S_t = 0$: $x_{t+1} = x_t - dx_t$

$$r_{t+1} = r_0 - cx_{t+1}$$

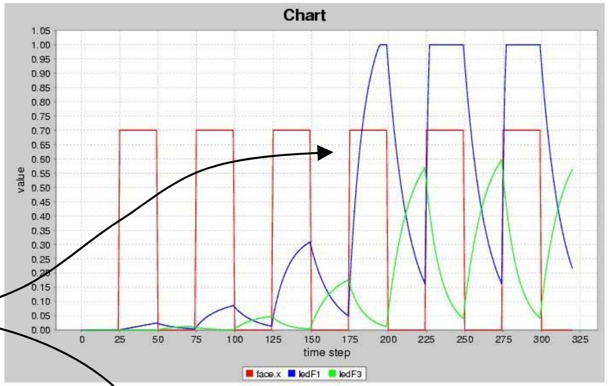
"Expectation" decreases when face is seen, but ***increases*** when face is lost

$$r_{0,n+1} = \left[\begin{array}{ll} r_{0,n} & S_{t+1} \le S_t \\ r_{0,n} + e \cdot r_t & Else, 0 < e < 1 \end{array}\right.$$

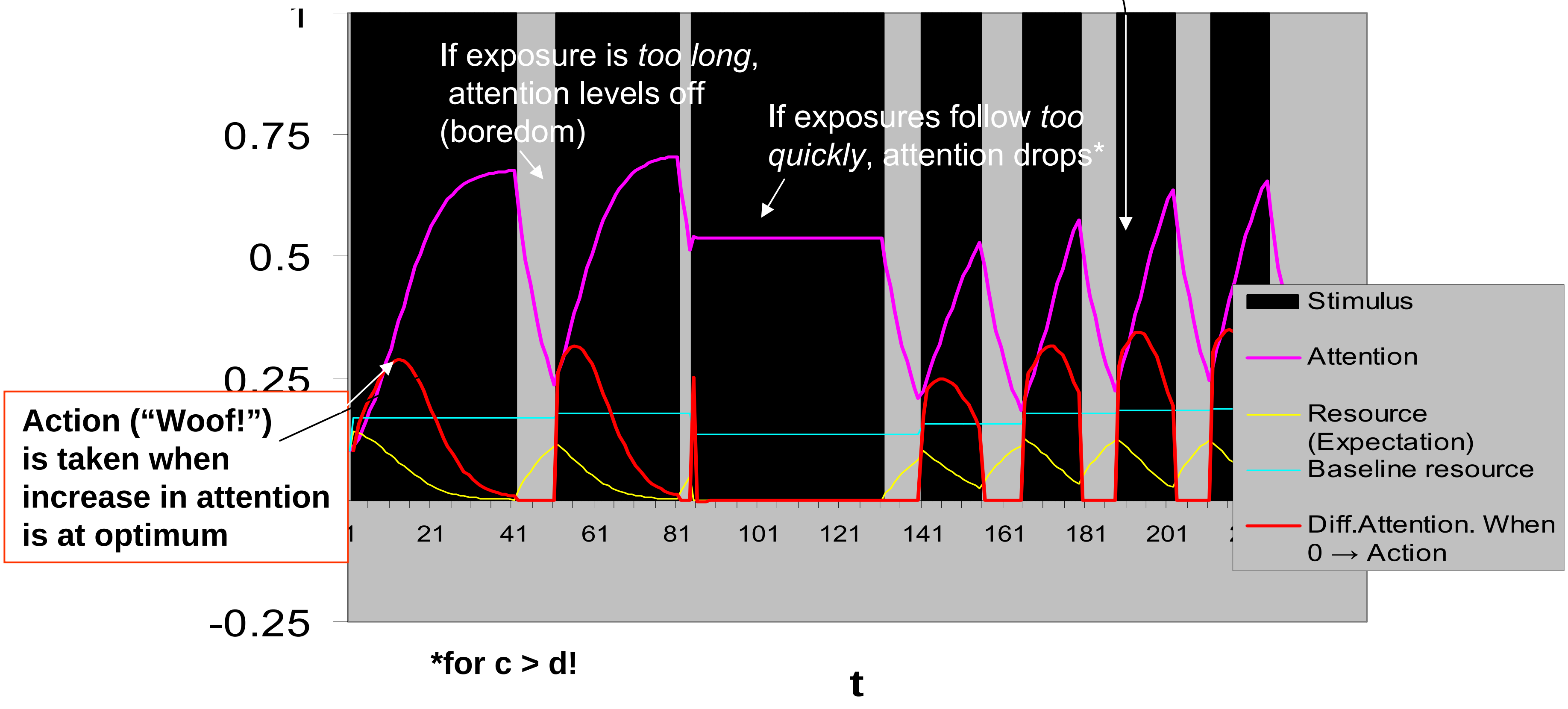
Updating of "expectation"

# RESULTS

**Original set-up (non-logistic):** exposures quickly following each other **boost** attention



## Additional features of logistic dynamics:



If exposure is *too long*, attention levels off (boredom)

If exposures follow *too quickly*, attention drops*

**Action ("Woof!") is taken when increase in attention is at optimum**

***for c > d!**

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***$r_t$ = Expectation***

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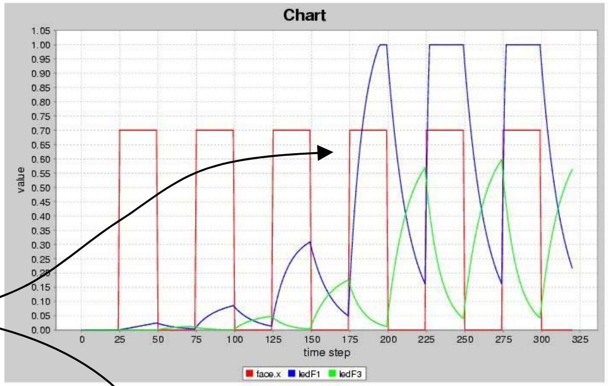

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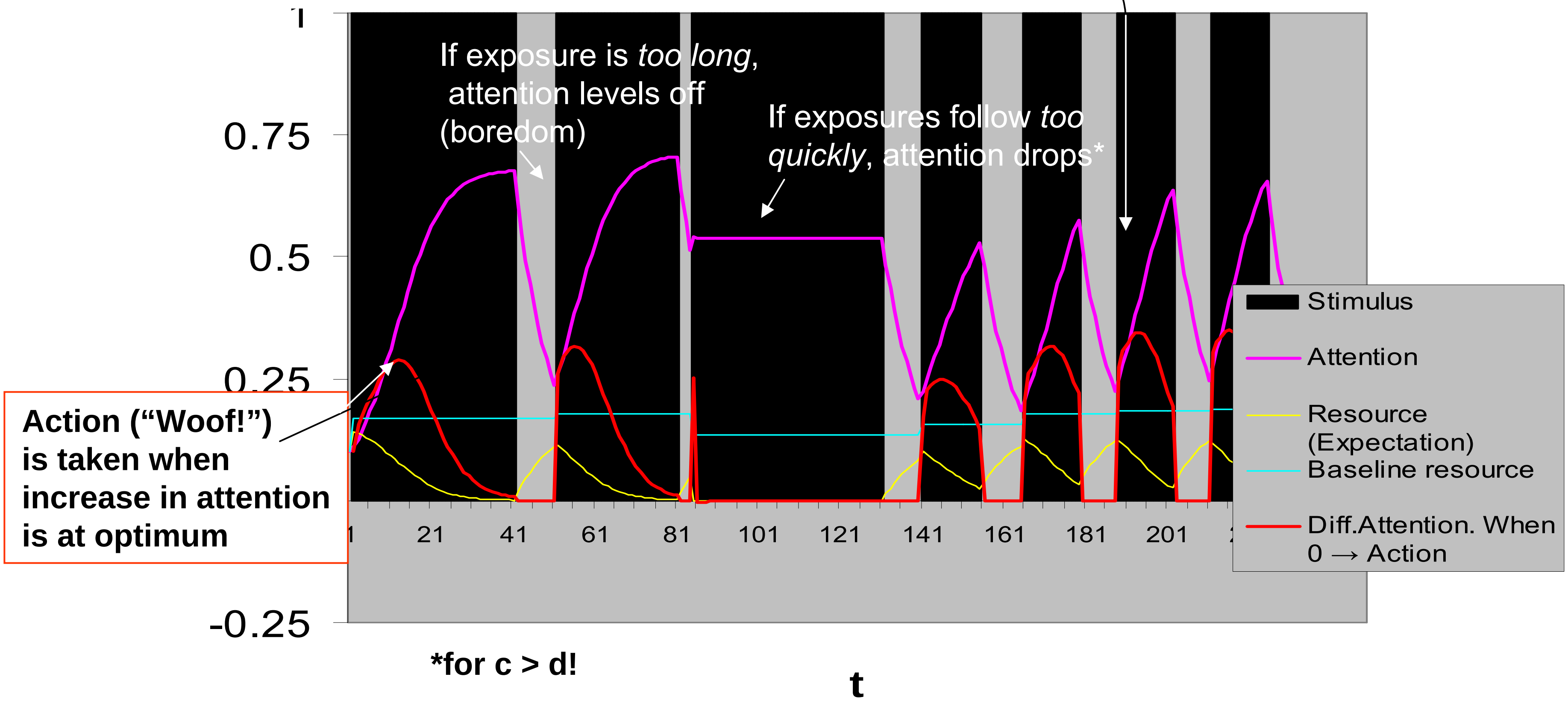

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**t**